Welcome to Mathematical Modelling FK (FRTN45)

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Department of Automatic Control, LTH

Outline

Thursday lecture

- Course introduction
- Models from physics (white boxes)

Friday lecture

- Models from data (black boxes)
- Mixed models (grey boxes)

Matematisk Modellering FK (FRTN45)

Course homepage:

`http://www.control.lth.se/course/FRTN45`

- 4.5 högskolepoäng ; betyg U/G
- 4 h lectures
- 100 h project

Project

- Project supervision from
  - Mathematics, Mathematical Statistics, Automatic Control.
- Project plan. An A4-paper prepared after consulting the supervisor. Send to course responsible by January 29. Use email with subject line "FRTN45".
- Written report
- Oral presentation (shared among all group members)
- Opposition (all team members together)
  Written opposition report
- 4 persons per project (possibly less)

Written report

See the website instructions:

- Cover sheet
- Summary
- Table of Contents
- Main Text
  - Presentation of problem: What is the purpose of the model?
  - Summary of used literature
  - Theory/Method
  - Implementation
  - Results
  - Evaluation/discussion: Does the model suit its purpose?
  - Reference list
- Description of how the work is distributed within the group
- Presentation of the course theory part

All Three Modelling Phases Must be Described

1. Problem structure
   - **Formulate purpose**, requirements for accuracy
   - Break up into subsystems — What is important?
2. Basic equations
   - Write down the relevant physical laws
   - Collect experimental data
   - Test hypotheses
   - Validate the model against fresh data
3. Model with desired features is formed
   - Put the model on suitable form.
     (Computer simulation or pedagogical insight?)
   - Document and illustrate the model
   - Evaluate the model: **Does it meet its purpose?**

Group 1: Vilken fågel sjunger?

Kan du skilja på en talgoxe och en gråsparv när du hör fågelsång utanför fönstret? "Kvitteromat" är en helt ny app som identifierar fågelarter baserat på sången. Den är dock, enligt utvärdering, känslig för störningar och ganska osäker i sitt beslut, då resultatet den presenterar är tre olika förslag på vilken art det är som sjunger. Detta projekts syfte är att identifiera några av våra vanligaste fågelarter genom att analysera deras sång, (lättare och svårare inspelningar), och hitta lämpliga kriterier för säker klassificering. Eventuellt kan en jämförelse och utvärdering göras mot kvitteromat. Verktyg för stationära stokastiska processer är användbara, tillsammans med information om signalens variation över tid. Data-material i mp3-format samt några mindre program för Matlab kommer att tillhandahållas.

Advisor: Maria Sandsten, MatStat
First meeting: Monday January 22 at 15.15 in advisors office
Final presentation: Tuesday March 6 at 13.15-17.00

Group 2-3: Statistical Analysis of fMRI data

The aim of the project is to analys fMRI data from a trial where the subject judged whether a pair of words rhymed or not. The experiment con- sisted of alternating 20-second work and rest blocks and we want to identify which parts of the brain that are active during the task. To illustrate possible techniques and to make the project feasible you will study a single slice from the fMRI (rather than the complete 3D voxel data). The slice consists of a video sequence with 160 time points representing the experiments 320s runtime (at 2s temporal resolution). Active pixels can be detected as those that exhibit a 10 sample ($10 \cdot 2 = 20$s) frequency.

Advisor: Johan Lindström, MatStat
First meeting: Monday January 22 at 15.15 in advisors office
Final presentation: Thursday March 1 at 13.15-17.00

Group 4-5: Teknologens väg till examen

Teknologens väg till examen är intressant att modellera för högskolorna, som kan använda modellen för att prediktera hur många teknologer som kommer att ta examen inom en viss tid. Vägen till examen kan ses som ett antal övergångar mellan diskreta tillstånd, såsom registrering till respektive programtermin, studieuppehåll, utbytesstudier eller inaktivitet, och avslutas med antingen examen eller studieavbrott. Målet med detta projekt är att modellera studievägen för LTH's teknologer utifrån terminsdata från LADOK, och utföra simulering av en årskull studenters väg genom utbildningssystemet.

Advisor: Mattias Fält, Reglerteknik
First meeting: Monday 22/1 at 15.15 in advisors office
Final presentation: Tuesday March 6 at 13.15-17.00

Group 6: Modellering av pappershelikoptrar

Med hjälp av en sax kan man ganska lätt bygga en liten pappershelikopter som när man släpper den faller och roterar. Ganska snabbt blir rotationshastigheten jämn. Projektet går ut på att försöka modellera detta förlopp, att mäta rotationshastigeterna experimentellt, t ex genom att filma fallet och att försöka utveckla en modell som kan förklara rotationshastigheten.

Advisor: Kalle Åström, Matematik LTH
First meeting: Monday 22/1 at 13.15 in advisors office
Final presentation: Tuesday March 6 at 13.15-17.00

Group 7: Structure and motion for sound

Using several microphones it is possible to calculate the position of sound sources. If the microphone positions are known this is usually called trilateration. If neither the sound sources nor the microphone positions are known, the problem is more challenging. The purpose of this project is to study and develop mathematical models for sound and use them in experiments with real data for structure and motion for sound. There is a choice to focus more on the signal processing for the sound or to focus on the geometrical aspects of the positions of the microphones and the sounds sources.

Advisor: Kalle Åström, Matematik LTH
First meeting: Monday 22/1 at 13.15 in advisors office
Final presentation: Tuesday March 6 at 13.15-17.00

Group 8: Modelling traffic flow of future vehicles

This project deals with the modelling of traffic flow of vehicles, all behaving the same, which is likely to be the case with future driver-less vehicles. With simple assumptions, one can study relations between the duration of green/red traffic lights, the traffic flux and vehicle density on a drive through. How can the traffic flow be controlled and optimized? Traffic flow is a nonlinear phenomenon: the flux (number of vehicles per time unit) on a drive through can be obtained with two different total times spent in a vehicle. What is best for the passengers, the environment? This project will give some basic knowledge of nonlinear modelling and the formation of shock waves, valuable for any modeller. Many time- and spatial-dependent physical phenomena are modelled with the conservation law of mass and the resulting governing equation is often a hyperbolic partial differential equation. This continuum approach is used for fluid flow but also for the flow of discrete particles.

Advisor: Stefan Diehl, Matematik LTH
First meeting: Friday 19/1 at 13.15 in advisors office
Final presentation: Thursday March 1 at 13.15-17.00

Group 9: Stokastisk populationsdynamik

Ett mycket viktigt inslag i populationsmodeller är sk Markov Jump processes. Intuitivt, kan de beskrivas som processer där för det mesta händer ingenting på mycket korta tidsintervall, men då det händer något, är effekten "dramatiskt" (exempelvis antalet friska i en population ändras med +1). Man kan beskriva sådana processer med den sk Kolmogorov Forward Equation (1931) och den första numeriska algoritmen som implementerar idén gjordes av Kendall (1950), efter forskning av Feller från 1940. Syftet är att applicera sådana modeller på en lagom komplicerad populationsdynamisk process.

Advisor: Mario Natiello, Matematik LTH
First meeting: Monday 22/1 at 13.15 in advisors office
Final presentation: Thursday March 1 at 13.15-17.00

Mathematical modelling

- Why modelling?
  - Natural sciences: Models for analysis (understanding)
  - Engineering sciences: Models for synthesis (design)
  - Specification: Model of a good technical solution
- Physical modeling (white boxes, **today!**)
  Model derived from fundamental physical laws
- Statistical methods (black boxes, **Friday's lecture**)
  Model derived from measurement data
  - Singular Value Decomposition (SVD)
  - Machine Learning
  - System Identification / Time Series Analysis
- Combination of the two (gray boxes)

Modelling in three phases:

1. Problem structure
   - **Formulate purpose**, requirements for accuracy
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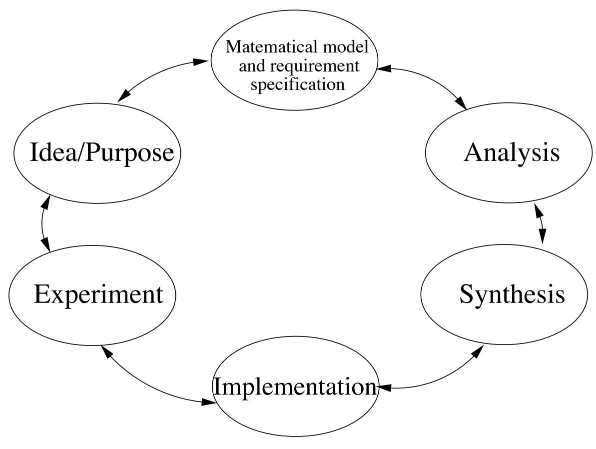
   - Document and illustrate the model
   - Evaluate the model: **Does it meet its purpose?**



Engineering Ethics [1]

- Relevant for the Pi-program?
- Ethical linear algebra?
- Ethical mathematical modelling?

[1] Thanks to Maria Henningsson Pi-02 for suggesting the next few slides.

"Our calculations show that..."

- What is behind the numbers?
- What assumptions are made?
- What limitations are there?

"Essentially, all models are wrong, but some are useful."
- George E. P. Box.

Knowledge Gives You Power and Responsibility

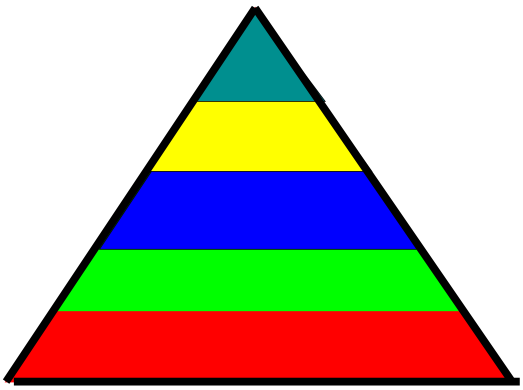
- Your expert role will give you an advantage
- What assumptions are made?
- What limitations are there?

Example1: The CitiCorp Building

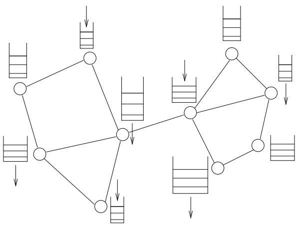
Example 2: The Parental Leave Insurance

What percentage of your income do you get?

Example 3: Mortage Securities



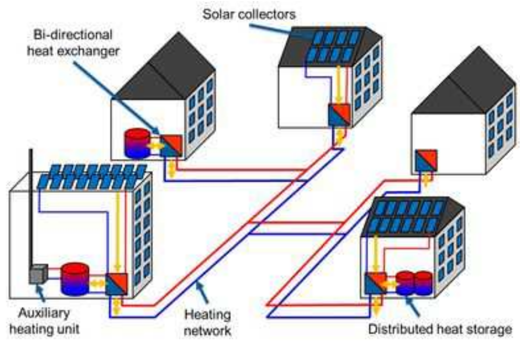
My Own Research: Dynamic Buffer Networks



- Producers, consumers and storages
- Examples: water, power, traffic, data
- Discrete/continuous, stochastic/deterministic
- Multiple commodities, human interaction

Problem: Scalable and adaptive methods for control.

Example: Ectogrid



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- Course introduction
- Models from physics (white boxes)
  - **Analogies and dimensions**
  - Modelica — A modern modeling language

Friday lecture

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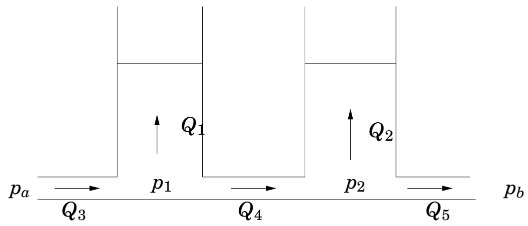
- Mixed models (grey boxes)

Principles and analogies: Hydraulics

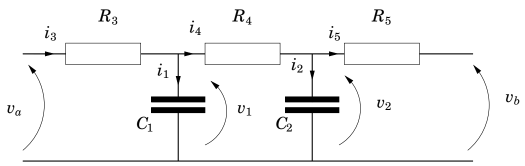
Example 1. A hydraulic system:



Incompressible fluid. Pressures: p_a, p_1, p_2, and p_3.
Volume flows: Q_1, Q_2, Q_3, Q_4, and Q_5.

Principles and analogies: Electrics

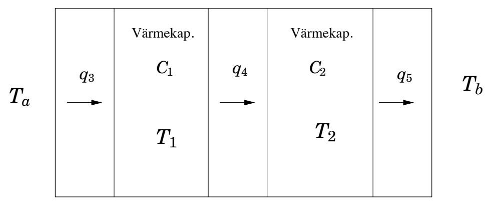
Example 2. An electrical system:



Potentials v_a , v_b , v_1, and v_2
Currents i_1 , i_2, i_3, i_4, and i_5

Principles and analogies: Heat

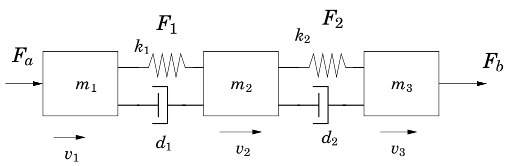
Example 3. A thermal system
(heat transfer through a wall):



Two elements with thermal capacities C_1 and C_2 separated by insulating layers. Heat flows: q_3 , q_4 and q_4
Temperatures: T_a , T_b , T_1 and T_2

Principles and analogies: Mechanics

Exempel 4. A mechanical system:



External forces: F_a and F_b
Velocities: v_1, v_2 and v_3
Spring constants: k_1 and k_2
Damping constants: d_1 and d_2

Analogies

Analogies: hydraulic - electric - thermal - mechanical
Two types of variables:

A. Flow Variables
- volume flow
- power flow
- heat flow
- speed

B. Intensity variables
- pressure
- voltage
- temperature
- force

For both of them addition rules hold.

Analogies (cont'd)

Intensity variations

$$C \cdot \frac{d}{dt}(\text{intensity}) = \text{flow}$$

C "capacitance":
hydraulic: $A/(\rho g)$
electrical: kapacitans
heat: thermal capacity
mechanical: inverse spring constant
Balance equations!

(More complicated if the capacitance is not constant.)

Analogies (cont'd)

Losses

$$\text{flow} = \phi(\text{intensity})$$

$$\text{intensity} = \varphi(\text{flow})$$

Hydraulic: flow resistance
Electrics: resistance
Heat: thermal conductivity
Mechanics: friction

Often linear relationship in the electrical case - nonlinearly in the other (may be approximated by linear for small changes of variables)

More phenomena

Intensity variations

$$L \cdot \frac{d}{dt}(\text{flow}) = \text{intensity}$$

L "inductance"
hydraucs: $\rho l/A$
electrics: inductans
heat: –
mechanics: mass
balance equations!

(more complicated if the inductance is not constant.)

Energy flows

Can you make a general modeling theory based on flow and intensity variables? Note the following.

pressure · flow	=	power
voltage difference · current	=	power
force · velocity	=	power
torque · angular velocity	=	power
temperature · heat flow	=	power · temperature

Dimension analysis

Physical variables have dimensions. E.g.,

$$[\text{density}] = ML^{-3}$$
$$[\text{force}] = M \cdot \frac{L}{T^2} = MLT^{-2}$$

where

$$M = [\text{mass}], \quad T = [\text{time}], \quad L = [\text{length}]$$

Physical connections must be dimensionally “correct”.

Example: Bernoulli’s law

In Bernoulli’s law $v = \sqrt{2gh}$ you have

$$[v/\sqrt{gh}] = LT^{-1}(LT^{-2}L)^{-0.5} = 1$$

$v/\sqrt{gh}$ is an example of dimensionless quantity.

Dimensionless quantities and scaling

Some historical passanger ships:

- Kaiser Wilhelm the great, 1898, 22 knots, 200 m
- Lusitania, 1909, 25 knots, 240 m
- Rex, 1933, 27 knots, 269 m
- Queen Mary, 1938, 29 knots, 311 m

Note that the ratio *(velocity)*^{2}*/(length)* is almost constant

Which physical phenomenon can be thought to be the cause?

2 min problem

Find the relationship (except for a scaling by a dimensionless constant) between a pendulum period time and its mass, its length and the acceleration of gravity g, i.e.,

$$t = f(m, l, g)$$

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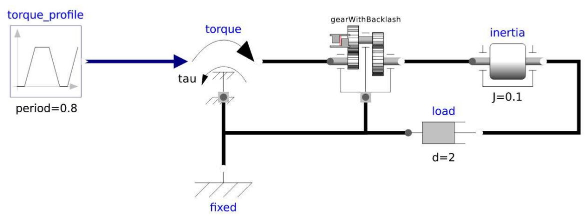
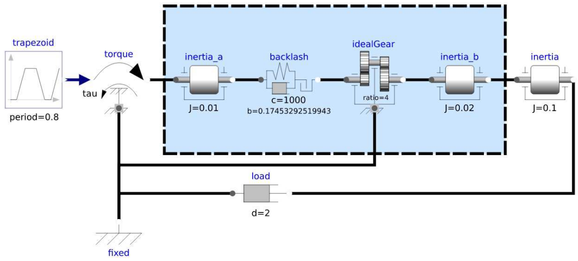
A Graphical Modelica Model



A submodel can be opened



Simple system in text format

```
model FirstOrder
  Real x;
equation
  der(x) = 1-x;
end FirstOrder;
```

Documentation is important:

```
model FirstOrderDocumented "A simple first order differential equation"
  Real x "State variable";
equation
  der(x) = 1-x "Drives value of x toward 1.0";
end FirstOrderDocumented;
```

Initialize at equilibrium!

```
model FirstOrderSteady
  "First order equation with steady state initial condition"
  Real x "State variable";
initial equation
  der(x) = 0 "Initialize the system in steady state";
equation
  der(x) = 1-x "Drives value of x toward 1.0";
end FirstOrderSteady;
```

A predator-prey model

$$\begin{cases} \dot{x} = x(\alpha - \beta y) \\ \dot{y} = y(\delta x - \gamma) \end{cases}$$

```
model ClassicModel "This is the typical equation-oriented model"
  parameter Real alpha=0.1 "Reproduction rate of prey";
  parameter Real beta=0.02 "Mortality rate of predator per prey";
  parameter Real gamma=0.4 "Mortality rate of predator";
  parameter Real delta=0.02 "Reproduction rate of predator per prey";
  parameter Real x0=10 "Start value of prey population";
  parameter Real y0=10 "Start value of predator population";
  Real x(start=x0) "Prey population";
  Real y(start=y0) "Predator population";
equation
  der(x) = x*(alpha-beta*y);
  der(y) = y*(delta*x-gamma);
end ClassicModel;
```

Re-using old models

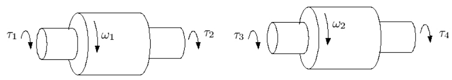
Inheritance:

```
model QuiescentModelWithInheritance "Steady state model with inheritance"
  extends ClassicModel;
initial equation
  der(x) = 0;
  der(y) = 0;
end QuiescentModelWithInheritance;
```

Modfication:

```
model QuiescentModelWithModifications "Steady state model with modifications"
  extends QuiescentModelWithInheritance(gamma=0.3, delta=0.01);
end QuiescentModelWithModifications;
```

Mathematics of general connection:



State models for two separate components:

$$\dot{\phi}_1 = \omega_1 \qquad \dot{\phi}_2 = \omega_2$$
$$J_1\dot{\omega}_1 = \tau_1 + \tau_2 \qquad J_2\dot{\omega}_2 = \tau_3 + \tau_4$$

Connection:

$$\phi_1 = \phi_2$$
$$\tau_2 = -\tau_3$$

The resulting model is not exactly a state model.

Linear differential-algebraic equations (DAE)

$$E\dot{z} = Fz + Gu$$

If E were non-singular, one could write

$$\dot{z} = E^{-1}Fz + E^{-1}Gu$$

which is a valid state model. If E is singular, variables have to be eliminated to get a state equation. Using a DAE solver is often better, since elimination can destroy sparsity.

Example:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & J_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\omega}_1 \\ \dot{\phi}_2 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \omega_1 \\ \phi_2 \\ \omega_2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix}$$

Nonlinear differential-algebraic equations (DAE)

Differential-algebraic equations, DAE

$$F(\dot{z}, z, u) = 0, \quad y = H(z, u)$$

u: input, y: output, z: "internal variable"

Special case: state model

$$\dot{x} = f(x, u), \quad y = h(x, u)$$

u: input, y: output, x: state