LUND INSTITUTE OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

FRTN20 – Market-driven Systems

Exam, 2013-08-26, 8.00 - 13.00

Points and grades

All answers must include a clear motivation. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem. Preliminary grade limits:

Grade 3: 12 points
4: 17 points
5: 22 points

Accepted Aids

Standard mathematical tables, authorized "Formelsamling i reglerteknik", and pocket calculator.

Exam Results

The result will be available no later than Friday June 8 2012. The results will be posted on the notice-board at the Department of Automatic Control, 1st floor M-building. The results as well as solutions will also be available on *http://www.control.lth.se/course/FRTN20*

1. Which production process type is primarily used for each of the following products (continuous, discrete, or batch)?

1. Soft drink
2. Airplane
3. Gasoline
4. Laptop
5. Table
6. Muffin

(2 p)

Solution

1. Batch, soft drinks are produced according to a recipe.
2. Discrete, airplanes consist of many discrete pieces which are put together.
3. Continuous, production of gasoline is open-ended.
4. Discrete, the components of the laptop are discrete pieces which are assembled.
5. Discrete, tables consist of several discrete pieces which are put together.
6. Batch, muffins are produced in batches according to a recipe.

2.

a. Describe at least two properties of a production rate. (1 p)

b. Describe what inventory variables are. Also give at least two examples. (1 p)

Solution

a. Some properties of the production rate are:

1. It is a single numeric value.
2. It is used to measure the speed of the production.
3. It can be used to detect production issues.

b. Inventory variables describe a stored property, for example liquid levels, gas pressure, and temperature.

3. Draw a batch process of each of the following types:

a) single-path, single-product

b) multi-path, single-product

c) multi-path, multi-grade

d) multi-path, multi-product

e) network, multi-grade

f) network, single-product

(3 p)

Solution

See Figure 1.

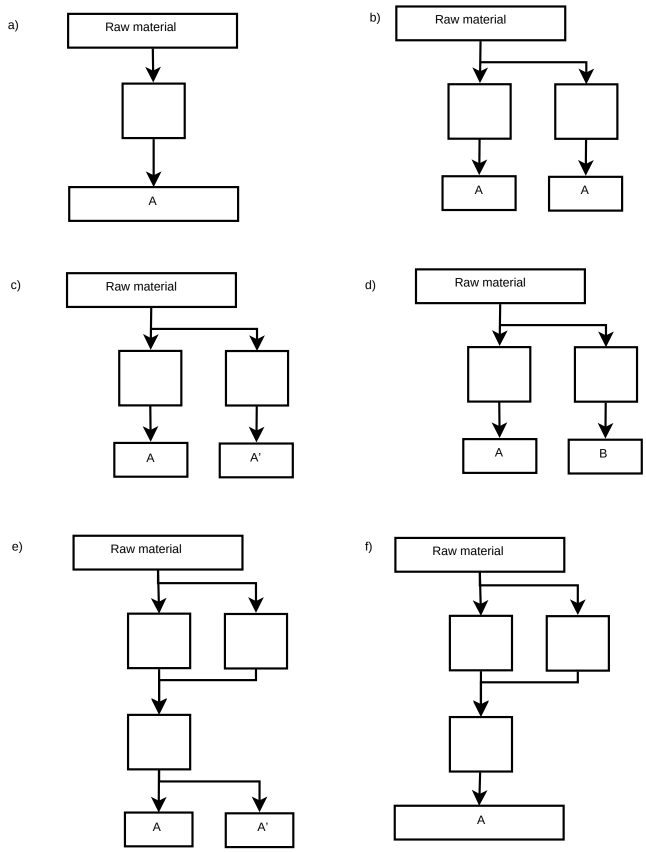

Figure 1 Batch processes of the right type. The products produced are A, B, and a variant of A called A'.

4. What is the recipe type for the recipes described by the following characteristics?

a) A generic recipe used for all batches in the production cell.

b) Only contains information about how the product is produced.

c) Not for a specific production cell. Adjusted with respect to the currently available raw materials.

d) Contains the measured oven temperature at time $t = 20s$.

e) Written in a customized language. Used in several production cells.

(2 p)

Solution

a) Master recipe

b) General recipe

c) Site recipe

d) Control recipe

e) Site recipe

5. A manufacturer produces two products, x_1 and x_2, with two machines, A and B. The cost of producing each unit of x_1 for machine A is 50 minutes, and for machine B is 30 minutes. The cost of producing each unit of x_2 for machine A is 24 minutes, and for machine B is 33 minutes. The working plans for a particular week are: 40 hours of work on machine A and 35 hours of work on machine B. Considering that the week starts with a stock of 30 units of x_1 and 90 of x_2, and a demand of 75 units of x_1 and 95 of x_2, plan the production in order to end the week with the maximum stock.

a. Write the problem on the canonical form given by

$$\begin{aligned} \max_x \quad & c^T x \\ \text{subject to} \quad & Ax \leq b, x \geq 0 \end{aligned}$$

(1 p)

b. Using the grid given in Figure 2 , draw the admissible region (2 p)

c. Solve the optimizacion problem (1 p)

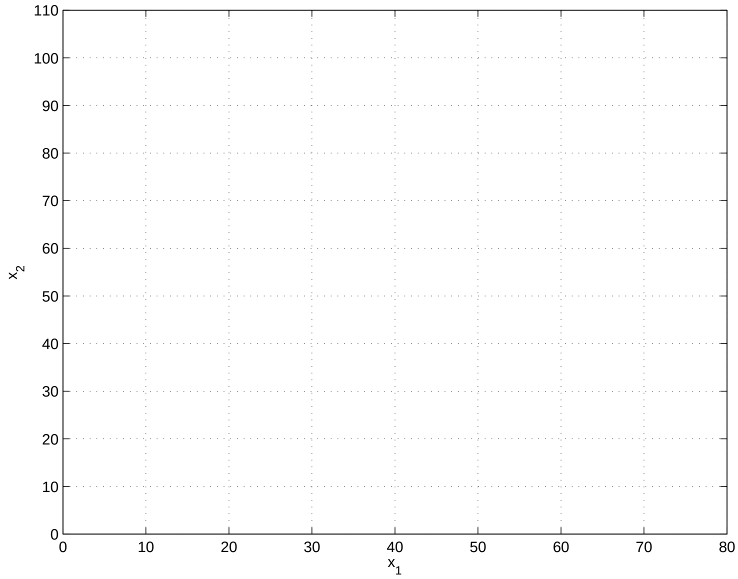

Figure 2 Admissible region.

Solution

a.

$$
\begin{aligned}
\max_{x_1,x_2} \quad & (x_1 + 30 - 75) + (x_2 + 90 - 95) \\
\text{subject to} \quad & 50x_1 + 24x_2 \leq 2400 \\
& 30x_1 + 33x_2 \leq 2100 \\
& x_1 \geq 75 - 30 \\
& x_2 \geq 95 - 90
\end{aligned}
$$

$$
\begin{aligned}
\max_{x_1,x_2} \quad & \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
\text{subject to} \quad & \begin{bmatrix} 50 & 24 \\ 30 & 33 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 2400 \\ 2100 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 35 \\ 5 \end{bmatrix}
\end{aligned}
$$

b. The admissible region is given by

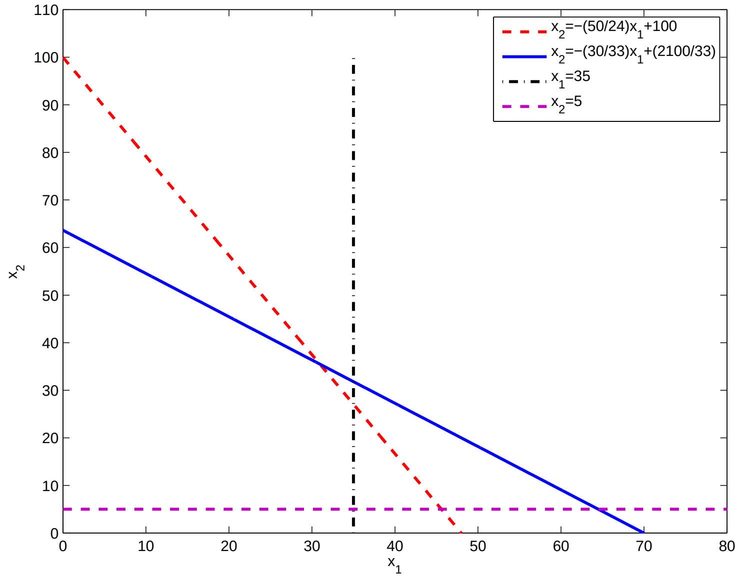

Figure 3 Admissible region.

c. The vertices of the admissible region have the following x_1, x_2 coordinates:

$$
v_0 = (35, 5), \quad v_1 = \left(\frac{456}{10}, 5\right), \quad v_2 = \left(35, \frac{650}{24}\right)
$$

Evaluating the objective $f(x_1, x_2) = x_1 + x_2 - 50$ on the vertices gives

$$
f(v_0) = -10, \quad f(v_1) = 0.6, \quad f(v_2) = 12.08
$$

Therfore the maximum occurs at v_2 which corresponds to $x_1 = 35$ and $x_2 = 650/24$.

6. A student is deciding what to purchase from a bakery. There are two choices of food: brownies, which cost 50 SEK each, and mini cheesecakes, which cost 80 SEK. The bakery is service-oriented and let the students purchase a fraction of an item if wished. The bakery requires 60 grams of chocolate to make each brownie (no chocolate is needed in the cheesecakes), 40 grams of sugar are needed for each brownie and 50 grams for each cheesecake. Finally, 40 grams of cream cheese are needed for each brownie and 120 grams for each cheesecake. Being health-conscious, the student has decided that he needs at least 120 grams of chocolate in his snack, along with 100 grams of sugar and 160 grams of cream cheese. He wishes to optimize his purchase by finding the least expensive combination of brownies and cheesecakes that meet these requirements.

a. Define the linear program and plot the admissible region. (2 p)

b. Formulate the dual problem and provide an interpretation of the dual variable. (2 p)

Solution

a. Let x_1 and x_2 be the number of brownies and mini cheesecakes respectively. The problem can be stated as follows:

$$\begin{aligned} \min_{x_1,x_2} \quad & 50x_1 + 80x_2 \\ \text{subject to} \quad & 60x_1 \geq 120 \\ & 40x_1 + 50x_2 \geq 100 \\ & 40x_1 + 120x_2 \geq 160 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The admissible region is given by

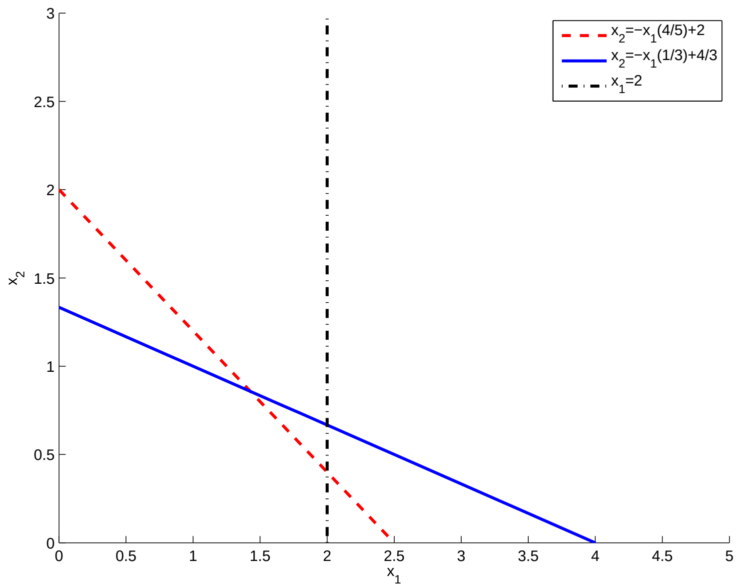

Figure 4 Admissible region.

b. The dual problem can be stated as follows:

$$\begin{aligned}
\max_{\lambda_1,\lambda_2,\lambda_3} \quad & 120\lambda_1 + 100\lambda_2 + 160\lambda_3 \\
\text{subject to} \quad & 60\lambda_1 + 40\lambda_2 + 40\lambda_3 \leq 50 \\
& 50\lambda_2 + 120\lambda_3 \leq 80 \\
& \lambda_1, \lambda_2, \lambda_3 \geq 0
\end{aligned}$$

The dual variables λ_1, λ_2 and λ_3 can be seen as the prices assigned to the chocolate, sugar and cream cheese by the bakery supplier in full knowledge of the student minimum nutritional requirements. knowledges

7. Production processes can be either discrete, batch or continuous. Describe four (4) characteristics for each production type. (3 p)

8. A worker and an employer have failed to agree on the salary S that the employer should pay the worker. An independent arbiter has been assigned to settle the dispute and determine S. The method will be the following: the employer and worker gives the arbiter final offers, e and w respectively. The arbiter then picks the offer closest to his predetermined idea of what a suitable salary is, lets call this number a (in case of a tie, the value $(e+w)/2$ will be used).

a. (1 p)

Assume the value a is known to both the worker and employer. Formulate the siutation as a zero-sum game and show that $(e^*, w^*) = (a, a)$ is a pure equilibrium. It is assumed that the worker wants to maximize the expected salary, and the employer wants to minimize it.

b. (2 p)

Assume now instead that a is unknown to the worker and employer and consider it a random variable with uniform distribution in the range $[L, H]$ where L and H are known. Determine all pure equilibria (e^*, w^*). You can assume that $L \leq e^* \leq w^* \leq H$.

Hint: Start by verifing that when $L \leq e \leq w \leq H$ the expected salary is

$$S(e,w) = \frac{1}{H-L}\left[\left(\frac{w+e}{2} - L\right)e + \left(H - \frac{w+e}{2}\right)w\right]$$

Solution

a. The game can be formulated as a zero-sum game were the employer is the minimizer and the worker the maximizer. The strategies are given by choices of e and w respectively and the payoff function is S. With $(e^*, w^*) = (a, a)$ we have $S(e^*, w^*) = a$. We also have $S(e^*, w) = S(e, w^*) = a$. Hence $S(e^*, w) \leq S(e^*, w^*) \leq S(e, w^*)$ which shows that it is a pure equilibrium.

b. We have

$$\left.\frac{\partial S}{\partial e}\right|_{e=e^*} = \frac{e^* - L}{H - L}$$

which is zero only when $e^* = L$. Similarly

$$\left.\frac{\partial S}{\partial w}\right|_{w=w^*} = \frac{H - w^*}{H - L}$$

which gives $w^* = H$. This proves that

$$(e^*, w^*) = (L, H)$$

is a (unique) equilibrium.

9. Find mixed Nash strategies for this non-zero sum two person game:

		Player 2 left	Player 2 right
Player 1	top	10,0	0,1
	bottom	0,1	10,0

The first number indicates the profit for player 1, the second number the profit for player 2. (2 p)

Solution

Assume x is the probability Player 1 chooses the top row and y the probability Player 2 chooses the left column. The profit for player 2 will be $1-x$ and x respectively. The best response for player 2 is hence "left" if $x < 1/2$ and "right" if $x > 1/2$ and indifferent if $x = 1/2$. Similar analysis for player shows that the best response for player 1 is "top" if $y > 1/2$ and "bottom" if $y < 1/2$ and indifferent if $y = 1/2$. The Nash strategy is given by the intersection and given by $(x, y) = (1/2, 1/2)$.