FRTF01 L9—Blood Flow Control

Lecture FRTF01 L9

- Blood Flow Modeling
- Blood Flow Control
- Blood Pressure Control
- Heart-Rate Control
- Temperature Control
- Acid-Base Control

Blood Flow Modeling

Windkessel model

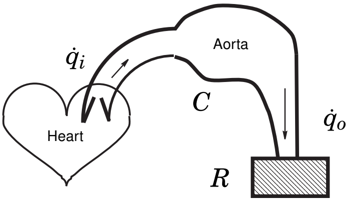
- Hemodynamics in major vessels;
- Arterial compliance—Elasticity, extensiblity
- Peripheral resistance—Resistance to flow through arterial tree
    - major arteries
    - minor arteries
    - arterioles
    - capillaries
- Pressure-flow relationship

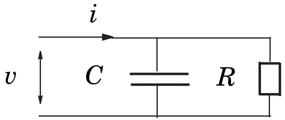
Blood Flow Modeling

Windkessel model—Two-element model WM2

- C Arterial compliance $C = dV/dp$
- R Peripheral resistance $R\dot{q}_o = p$
- p Blood pressure in aorta
- $\dot{q}$ Blood flow

$$C\frac{dp}{dt} + \frac{1}{R}p = \dot{q}_i \tag{1}$$





Blood Flow Modeling

- Input—Aortic blood inflow $\dot{q}_i$
- Output—Aortic blood pressure p

State-space model

$$\frac{dp}{dt} = -\frac{1}{RC}p + \frac{1}{C}\dot{q}_i \qquad (2)$$

Transfer function

$$\frac{P(s)}{\dot{Q}(s)} = \frac{1/C}{s + 1/RC} = \frac{R}{RCs + 1} \qquad (3)$$

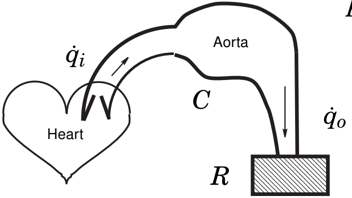
Blood Flow Modeling

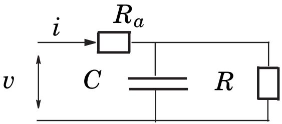
Windkessel model—Three-element model WM3

- C Arterial compliance $C = dV/dp$
- R Peripheral resistance $R\dot{q}_o = p$
- R_a Aortic resistance
- p_a, p Blood pressure in aorta proximally and distally, resp.
- $\dot{q}$ Blood flow; Inflow $\dot{q}_i$, Outflow $\dot{q}_o$

$$C\frac{dp}{dt} + \frac{1}{R}p = \dot{q}_i \qquad (4)$$

$$p_a = p + R_a\dot{q}_i \qquad (5)$$





Blood Flow Modeling

$\dot{q}_i$ Input—Aortic blood inflow

p_a Output—Aortic blood pressure

Differential equation

$$C\frac{dp}{dt} + \frac{1}{R}p = \dot{q}_i \qquad (6)$$

$$p_a = p + R_a\dot{q}_i \qquad (7)$$

State-space model

$$\frac{dp}{dt} = -\frac{1}{RC}p + \frac{1}{C}\dot{q}_i \qquad (8)$$

$$p_a = p + R_a\dot{q}_i \qquad (9)$$

Transfer function

$$\frac{P(s)}{\dot{Q}(s)} = \frac{1/C}{s+1/RC} + R_a = \frac{R}{RCs+1} + R_a = \frac{R_aRCs + R + R_a}{RCs+1}$$

Blood Flow Modeling

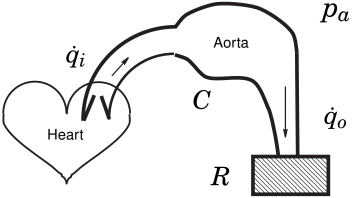
Windkessel model—Four-element model WM4

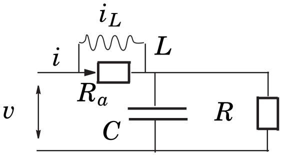
- R_a Aortic resistance
- L Inertia coefficient of blood flow
- p_a, p Blood pressure in aorta proximally and distally, resp.
- $\dot{q}$ Blood flow; Inflow $\dot{q}_i$, Outflow $\dot{q}_o$, Inertia flow $\dot{q}_L$

$$C\frac{dp}{dt} + \frac{1}{R}p = \dot{q}_i \tag{10}$$

$$L\frac{d\dot{q}_L}{dt} + R_a\dot{q}_L = R_a\dot{q}_i \tag{11}$$

$$p_a = p + R_a i - R_a\dot{q}_L \tag{12}$$





Blood Flow Modeling

$\dot{q}_i$ Input—Aortic blood inflow
p_a Output—Aortic blood pressure

Differential equation

$$
\begin{aligned}
C\frac{dp}{dt} + \frac{1}{R}p &= \dot{q}_i && (13)\\
L\frac{d\dot{q}_L}{dt} + R_a\dot{q}_L &= R_a\dot{q}_i && (14)\\
p_a &= p + R_a\dot{q}_i && (15)
\end{aligned}
$$

State-space model

$$
\begin{aligned}
\frac{dp}{dt} &= -\frac{1}{RC}p + \frac{1}{C}\dot{q}_i && (16)\\
\frac{d\dot{q}_L}{dt} &= -\frac{R_a}{L}\dot{q}_L + \frac{R_a}{L}\dot{q}_i && (17)\\
p_a &= p + R_a\dot{q}_i - R_a\dot{q}_L && (18)
\end{aligned}
$$

Blood Flow Modeling

State-space model

$$\frac{dp}{dt} = -\frac{1}{RC}p + \frac{1}{C}\dot{q}_i \quad (19)$$

$$\frac{d\dot{q}_L}{dt} = -\frac{R_a}{L}\dot{q}_L + \frac{R_a}{L}\dot{q}_i \quad (20)$$

$$p_a = p - R_a\dot{q}_L + R_a\dot{q}_i \quad (21)$$

or

$$\frac{d}{dt}\begin{bmatrix} p \\ \dot{q}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & -\frac{R_a}{L} \end{bmatrix}\begin{bmatrix} p \\ \dot{q}_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{R_a}{L} \end{bmatrix}\dot{q}_i \quad (22)$$

$$p_a = \begin{bmatrix} 1 & -R_a \end{bmatrix}\begin{bmatrix} p \\ \dot{q}_L \end{bmatrix} + R_a\dot{q}_i \quad (23)$$

Transfer function

$$\frac{P(s)}{\dot{Q}(s)} = \frac{R}{RCs+1} - \frac{R_a}{\frac{L}{R_a}s+1} + R_a = \frac{R}{RCs+1} + \frac{Ls}{\frac{L}{R_a}s+1}$$

Navier-Stokes Equations

State variables

- Pressure p [N/m^2]
- Velocity v [m/s]

Parameters

- Viscosity η [Ns/m^2]
- Acceleration of gravity [m/s^2]

Navier-Stokes equations

$$\begin{aligned}
\overbrace{\rho \frac{\partial v}{\partial t} + \rho (v \cdot \nabla) v}^{\text{Fluid inertia}} &= \overbrace{\nabla \cdot (\eta (\nabla v + (\nabla v)^T))}^{\text{Viscous friction}} + \overbrace{\rho g}^{\text{Gravity}} \overbrace{- \nabla p \; [\mathrm{N/m^3}]}^{\text{Pressure gradient}} \\
\underbrace{\nabla \cdot v}_{\text{Incompressibility}} &= 0 \; [\mathrm{s}^{-1}]
\end{aligned} \quad (24)$$

Blood Flow Control

Blood Flow Control

- Hemodynamics
- Hydrostatic pressure
- Viscosity
- Conservation of mass
- Peripheral resistance—Vasodilatation, vasocontriction
- Vessel compliance

Hemodynamics

Hemodynamics

Cardiac Output

- Stroke Volume (SV) = EDV − ESV
- Ejection Fraction (EF) = (SV / EDV) × 100%
- Cardiac Output (Q) = SV × HR
- Cardiac Index (CI) = Q / BSA = SV × HR/BSA

where

- HR is Heart Rate, expressed as BPM (Beats Per Minute)
- BSA is Body Surface Area [m^2].

Don't use the acronyms in mathematical expressions

Blood Flow Control

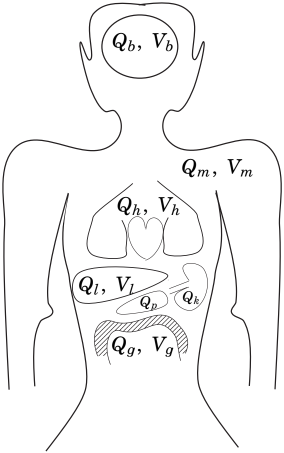
Regional blood flow—Mechanisms

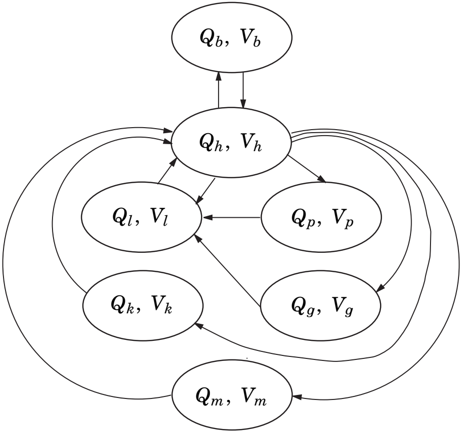
- Neural—Autonomous nervous system (ANS); Sympathetic
- Myogenic—
  - Increased pressure elicits vasoconstriction
  - Decreased pressure elicits vasodilatation
- Metabolic
  - Decrease in P_{O_2} or pH causes vasodilatation
  - Higher $[K^+]_{\mathrm{ECF}}$ causes vasodilatation
- Endothelial—Release of vasoactive substances

Blood Flow Control

- Local control—Tissue need for blood perfusion;
- Nervous control
  - Shift of blood to muscles during exercise;
  - Shift of blood to skin to control temperature;
- Humoral control—Local regulation in tissue flow

Body-Compartment Modeling





Compartment volumes $V_b, Vm, V_h, V_l, V_g, V_k, V_p$

Blood Flow Control

Cerebral blood flow—Vasodilatation

- $[H^+]$
- $[CO_2]$

Kidney blood flow—Constant flow independent of arterial pressure

- $[Na^{+2}]$
- [protein metabolites]

Muscle blood flow—Response to arterial pressure

- Linear
- Saturation—Arterial pressure $p \in [1, 2.5]$ [kPa]
- Linear

Blood Flow Control—Humoral Regulation

Humoral Regulation—Regulation by substances, e.g., hormones, ions

- Norepinephrine—Vasoconstrictor
- Epinephrine—Vasoconstrictor; Vasodilatation in skeletal and cardiac muscle;
- Angiotensin—Vasoconstrictor;
- Bradykinin—Vasodilatation;
- Vasopressin—Vasoconstrictor on arterioles; No effect on veins;
- Serotonin—Vasodilatation, Vasoconstriction;
- Histamin—Vasodilatation; Increase of capillary porosity;
- Prostaglandins—Vasodilatation, Vasoconstriction.

Blood Pressure Control—Why Pressure?

Poiseuille law

- Poiseuille law

$$\dot{V} = F = p \cdot \frac{\pi r^4}{8\eta l}, \qquad R = \frac{8\eta l}{\pi r^4} \tag{25}$$

η Viscosity [Ns/m]
l Length
r Inner radius [m]
p Pressure
R Resistance

Gravity

- Mean aortic pressure $p = 95$ [mm Hg]
- Hydrostatic pressure from erect posture

Conversion factor: 1 [mm Hg]= 133.32 [Pa]; [Pa]=[N/m^2]

Blood Pressure Control—Why Pressure?

Laplace law

$$T = p \cdot r \tag{26}$$

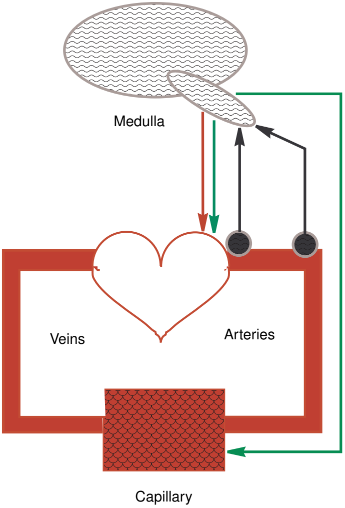
- T Vessel wall tension
- p Intravascular pressure – Tissue pressure
- r Vessel radius

Blood Pressure Control—Baroreflex

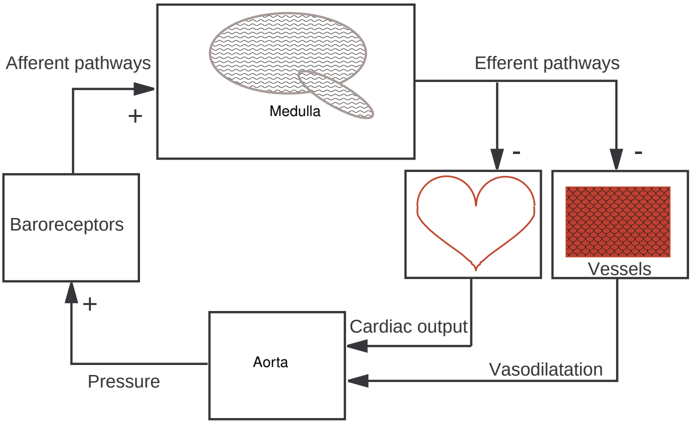
Baroreceptor loop

- Baroreceptors (pressoreceptors)
  - Carotid sinus
  - Aortic arch
- Baroreceptor afferents to CNS (medullary vasomotor center)—CN IX
- Sympathetic efferents
  - Stimulation of heart—rate and stroke volume
  - Constriction of veins
  - Contraction of arterioles and small arteries
- Parasympathetic efferents—n. vagus CN X
  - Decreasing heart rate
  - Decreasing strength of contraction

Blood Flow Control—Baroreflex



Blood Flow Control—Baroreflex



Heart-Rate Control

Anatomy

- Autonomic reflex control in medulla oblongata
- Receptors of carotid and aortic sinuses and bodies
- Cardiac innervation
  - + Adrenergic sympathetic nerves
  - – Dorsal motor nucleus of n. vagus

Temperature control—Homeothermy

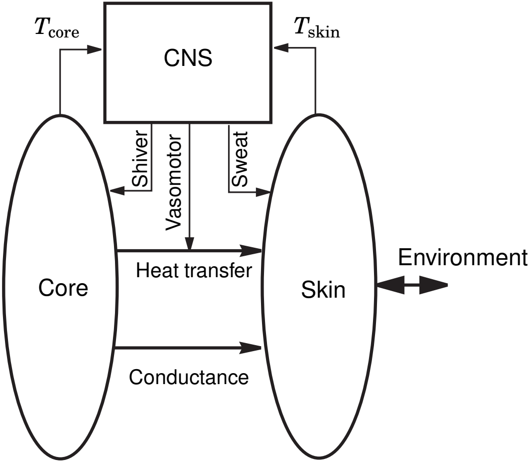
Heat balance

- $+$ Metabolism
- $-$ Work done on environment
- $-$ Heat loss
  - Convective
  - Evaporative
  - Radiative
  - Conduction
- $\sum$ Heat storage

- Resting metabolic rate (RMR): 85 [W] (72 [kcal/h])
- O_2 consumption $\dot{V}_{O_2}$: 250 [mL/min]

Conversion factor: 1 kcal/h = 1.162 W; [W]=[J/s]=[Nm/s]

Temperature control—Homeothermy



Acid-Base Control

Regulation of $[H^+]$—pH control

- Acidosis—Dying in coma
- Alkalosis—Dying in tetany or convulsions

Normal pH

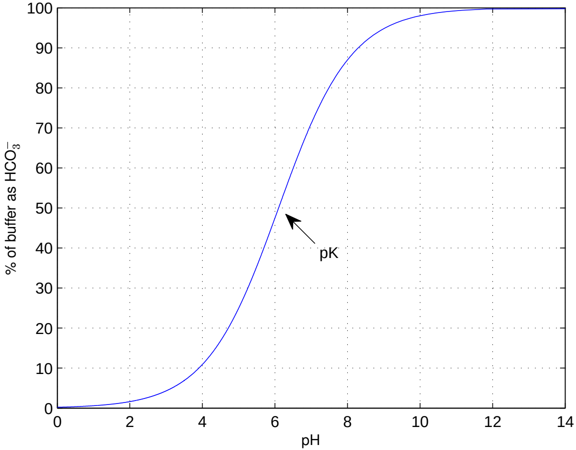
- Arterial blood: pH=7.4
- Venous blood, ISF: pH=7.35
- Survival range, blood: pH$\in$ [7.0, 7.7]
- ICF: pH=7.0, pH$\in$ [4.5, 7.4]
- Gastric secretion: pH$\in$ [1.0, 3.5]
- Pancreatic secretion: pH$\in$ [8.0, 8.3]

Regulation of pH

- Buffer systems
- Renal excretion of H^+—Acid or alcaline urine
- Respiratory excretion of CO_2

Acid-Base Control

Bicarbonate buffer



Transportation of $[H^+]$ against gradient

Acid-Base Control

Buffer systems

- Bicarbonate buffer pK=6.1
  - – pH=7.4 far from pK
  - – Low concentrations
- Elimination of strong acids and bases
  - HCl+ $NaHCO_3 \rightarrow H_2CO_3$+NaCl
  - NaOH + $H_2CO_3 \rightarrow NaHCO_3+H_2O$
- Phosphate buffer pK=6.8
  - + pH=7.4 close to pK—Efficient buffering
  - – Low concentrations
- Protein buffer pK:s ≈ 7.4
  - + pH=7.4 close to pK:s—Efficient buffering
  - + Big buffer
- Ammonia buffer

Acid-Base Control

Respiratory feedback

- Feedback of pH
- Feedback gain $K = 1 - 3$, $\tau > 1$ [min] (Guyton p.491)

How efficient is the respiratory compensation?

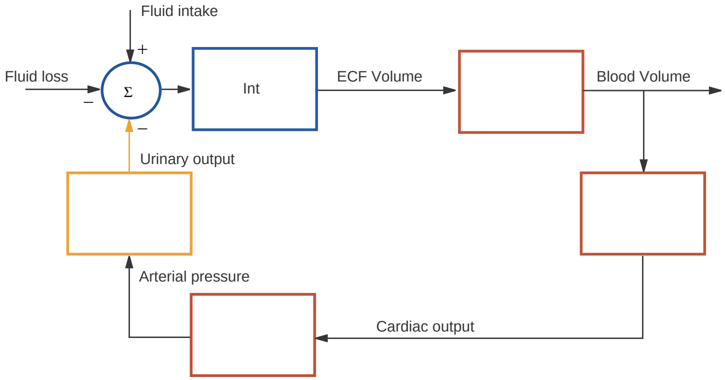
Hint: Calculate sensitivity function or make a step response!

Acid-Base Control

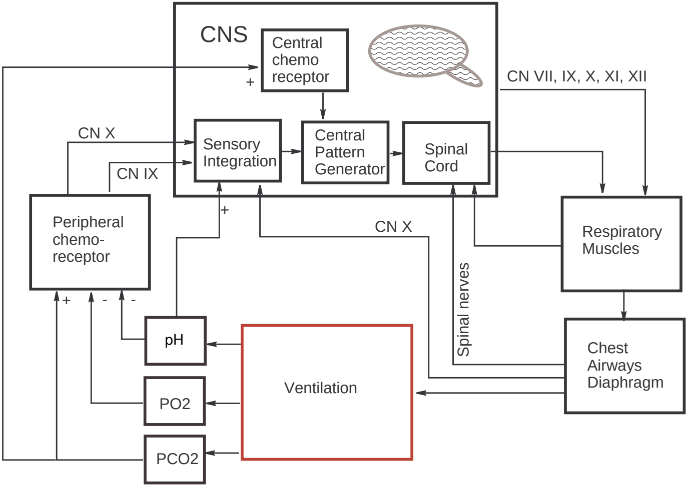
Renal regulation

- Elimination of H^+; Reabsorption of Na^+
- Excretion in proximal tubules
- Urine pH$\in$ [4.5, 8.0]
- Capacity 500 [mmol/day]

Regulation of Blood Volume



Control of Ventilation



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- A. C. Guyton, *Textbook of Medical Physiology (5th Ed.)*, W. B. Saunders, Philadelphia, PA, 1976.