FRTN15 Predictive Control - Exercise Session 5
Model Reference Adaptive Systems

1. With the controller structure

$$u = \theta u_c$$

the transfer function from command signal to the output becomes $\theta k G(s)$. This transfer function is equal to $G_m(s)$ if the parameter θ is chosen as

$$\theta = \frac{k_0}{k}$$

We will now use the MIT-rule to obtain a method for adjusting the parameter θ when k is not known. The error is

$$e = y - y_m = kG(p)\theta u_c - k_0 G(p) u_c$$

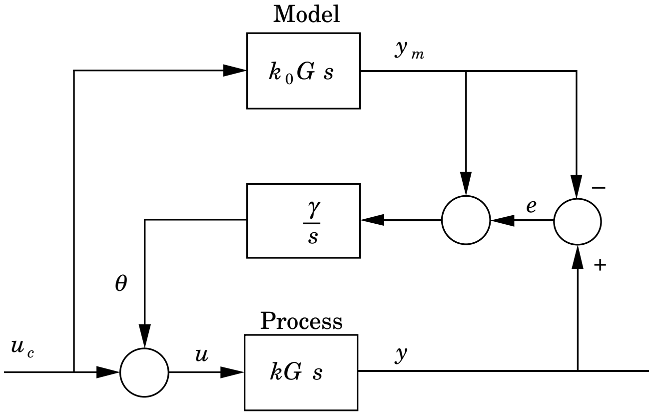
where u_c is the command signal, y_m the model output, y the process output, θ the adjustable parameter, and $p = d/dt$ the differential operator. The sensitivity derivative is given by

$$\frac{\partial e}{\partial \theta} = kG(p)u_c = \frac{k}{k_0} y_m$$

The MIT rule then gives the following adaptation law

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_0} y_m e = -\gamma y_m e \tag{1}$$

where $\gamma = \gamma' k/k_0$ has been introduced instead of γ'. Notice that in order to have the correct sign of γ it is necessary to know the sign of k. Equation 1 gives the law for adjusting the parameter. A block diagram of the system is shown in Figure 1.



Figur 1 Block diagram of an MRAS for adjustment of a feedforward gain based on the MIT rule.

2. The PI version of the SPR rule is

$$\frac{d\theta}{dt} = -\gamma_1 \frac{d}{dt}(u_c e) - \gamma_2 u_c e \tag{2}$$

To derive the error equation we notice that

$$\frac{dy_m}{dt} = \theta^0 u_c$$
$$\frac{dy}{dt} = \theta u_c$$

Hence

$$\frac{de}{dt} = (\theta - \theta^0)u_c$$

we get

$$\frac{d^2e}{dt^2} = \frac{d\theta}{dt}u_c + (\theta - \theta^0)\frac{du_c}{dt}$$

Inserting the parameter updata law from Equation 2 into this we get

$$\frac{d^2e}{dt^2} = -\gamma_1\left(\frac{du_c}{dt}e + u_c\frac{de}{dt}\right)u_c - \gamma_2 u_c^2 e + (\theta - \theta^0)\frac{du_c}{dt}$$

Hence

$$\frac{d^2e}{dt^2} + \gamma_1 u_c^2\frac{de}{dt} + \left(\gamma_1 u_c\frac{du_c}{dt} + \gamma_2 u_c^2\right)e = (\theta - \theta^0)\frac{du_c}{dt}$$

Assuming that u_c is constant we get the following error equation

$$\frac{d^2e}{dt^2} + \gamma_1 u_c^2\frac{de}{dt} + \gamma_2 u_c^2 e = 0$$

Assuming that we want this to be a second order system with ω and ζ we get

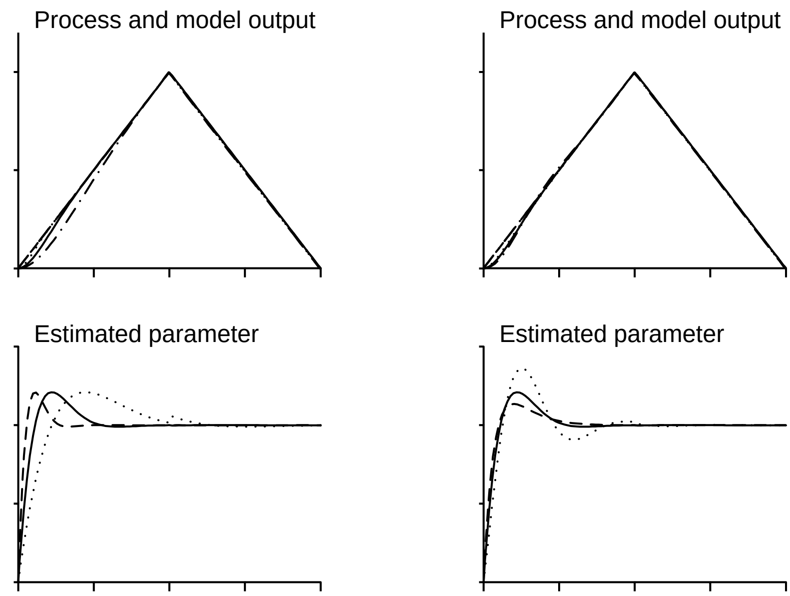
$$\begin{cases}\gamma_1 u_c^2 = 2\zeta\omega \\ \gamma_2 u_c^2 = \omega^2\end{cases} \qquad \begin{array}{l}\gamma_1 = 2\zeta\omega/u_c^2 \\ \gamma_2 = \omega^2/u_c^2\end{array}$$

This gives an indication of how the parameters γ_1 and γ_2 should be selected. The analysis was based on the assumption that u_c was constant. To get some insight into what happens when u_c changes we will give a simulation where u_c is a triangular wave with varying period. The adaptation gains are chosen for different ω and ζ. Figure 2 shows what happens when the period of the square wave is 20 and $\omega = 0.5$, 1 and 2. Corresponding to the periods 12, 6 and 3. Figure 3 show what happen when u_c is changed more rapidly.

3. The controller has two parameters. If they are chosen as

$$\theta_1 = \theta_1^0 = \frac{b_m}{b} \tag{3}$$
$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b} \tag{4}$$

the input-output relations of the system and the model are the same. This is called perfect model following.



Figur 2 Simulation in Problem 5.2 for a triangluar wave of period 20. Left top: Process and model outputs, Left bottom: Estimated parameter θ when $\omega = 0.5$ (full), 1 (dashed), and 2 (dotted) for $\zeta = 0.7$. Right top: Process and model outputs, Right bottom: Estimated parameter θ when $\zeta = 0.4$ (full), 0.7 (dashed), and 1.0 (dotted) for $\omega = 1$.

To apply the MIT rule, introduce the error

$$e = y - y_m$$

where y denotes the output of the closed-loop system. The closed loop system is given by:

$$y = \frac{b\theta_1}{p + a + b\theta_2} u_c$$

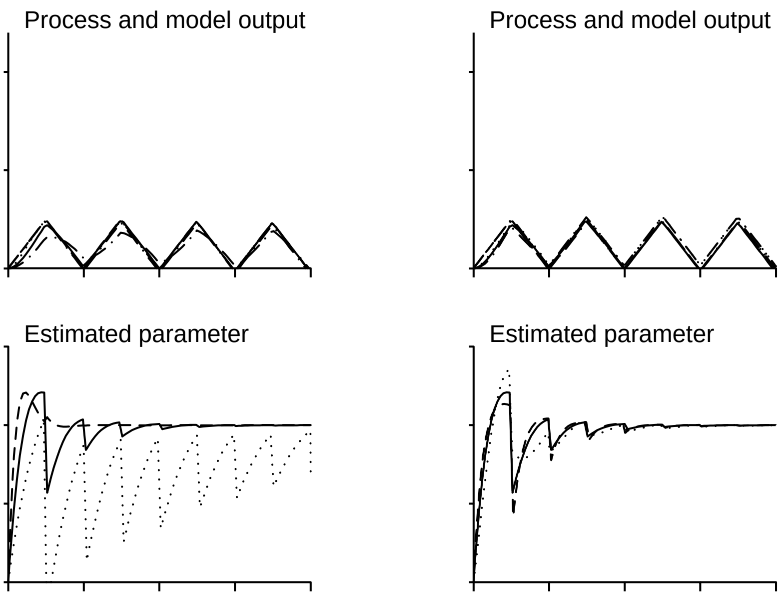
where $p = d/dt$ is the differential operator. The sensitivity derivatives are obtained by taking partial derivatives with respect to the controller parameters θ_1 and θ_2

$$\begin{aligned}
\frac{\partial e}{\partial \theta_1} &= \frac{b}{p + a + b\theta_2} u_c \\
\frac{\partial e}{\partial \theta_2} &= -\frac{b^2\theta_1}{(p + a + b\theta_2)^2} u_c = -\frac{b}{p + a + b\theta_2} y
\end{aligned}$$

These formulas cannot be used directly, because the process parameters a and b are not known. Approximations are therefore required. One possible approximation is based on the observation that $p + a + b\theta_2^0 = p + a_m$ when the parameters give perfect model following. We will therefore use the approximation

$$p + a + b\theta_2 \approx p + a_m$$

which will be reasonable when parameters are close to their correct values. With this approximation we get the following equations for updating the



Figur 3 Simulation in Problem 5.2 for a triangluar wave of period 5. Left top: Process and model outputs, Left bottom: Estimated parameter θ when $\omega = 0.5$ (full), 1 (dashed), and 2 (dotted) for $\zeta = 0.7$. Right top: Process and model outputs, Right bottom: Estimated parameter θ when $\zeta = 0.4$ (full), 0.7 (dashed), and 1.0 (dotted) for $\omega = 1$.

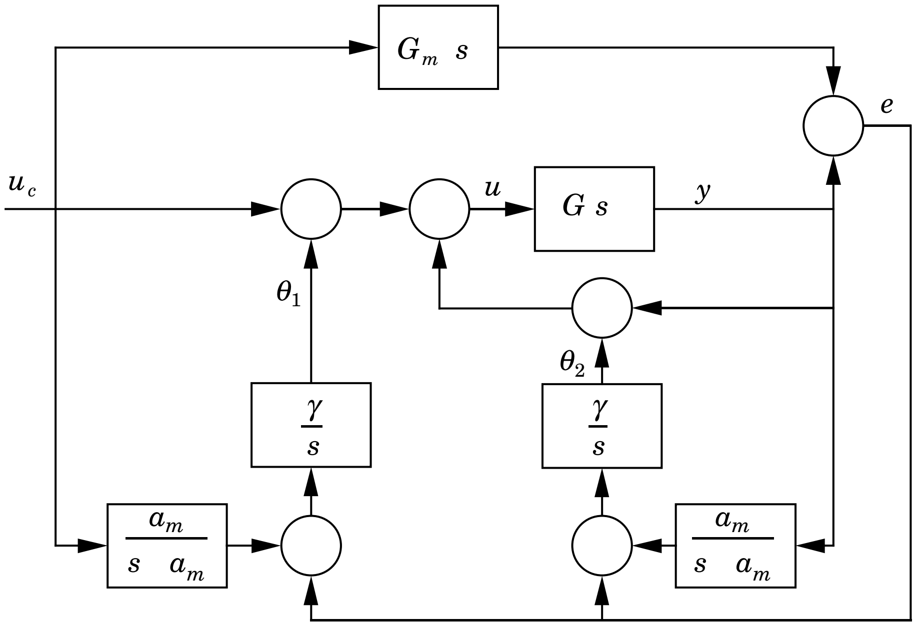
controller parameters:

$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{p + a_m} u_c \right) e \tag{5}$$

$$\frac{d\theta_2}{dt} = \gamma \left(\frac{a_m}{p + a_m} y \right) e \tag{6}$$

In these equations we have combined parameters b and a_m with the adaptation gain γ', since they appear as the product $\gamma' b / a_m$. The sign of parameter b must therefore be known in order to have the correct sign of γ.

A block diagram of the resulting system is shown in Figure 4.



Figur 4 Block diagram of a model reference controller for a first order process