

# FRTN15 Predictive Control—Home Work 1

## Signals and Systems

In this homework exercise we recapitulate theory for discrete time signals and systems in assignments 1-3. Recursive Least square estimation (RLS) is treated in assignment 4. The exercise also gives the opportunity to practice Matlab/Simulink.

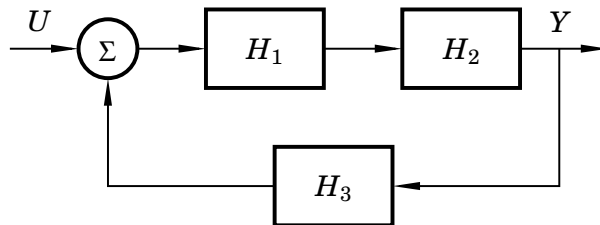
**E-mail your detailed and motivated solutions in pdf-format to [jerker@control.lth.se](mailto:jerker@control.lth.se). Attach any Matlab code or Simulink models you might have used.**

1. Sample the (continuous time) system

$$G_{y,u}(s) = \frac{s+1}{s^2}.$$

Provide the transfer function and a state space representation. Conduct the calculations

- a. by hand, with a parametrized sample period  $h$ .
  - b. using matlab, with sample period  $h = 0.1$ .
  - c. How does the sampling period affect the result?
2. Give the transfer function  $H(z)$  from  $u$  to  $y$  of the interconnection below, where  $H_1(z) = z + 2$ ,  $H_2(z) = \frac{1}{z^2+2z+1}$  and  $H_3(z) = \frac{z}{z+2}$ . Also plot the response  $y$  when  $u$  is a step function.



3.
  - a. Show that the system  $H$  given by

$$x(kh + h) = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} \frac{h^2}{2} \\ h \end{pmatrix} u(kh)$$

$$y(kh) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(kh)$$

is not asymptotically stable. Here  $h > 0$  denotes the sampling period.

- b. Stabilize  $H$  by means of linear state feedback. The resulting system should have all eigenvalues placed in the origin; use  $h = 0.1$ . (Hint: The command `acker` in the matlab control system toolbox can be useful.)

- c. Placing all eigenvalues in the origin is referred to as deadbeat control. Give an interpretation, motivating this nomenclature. What potential drawbacks are there, compared to less aggressive eigenvalue placements? Hint: Study the size of  $K$  when  $h$  approaches zero.

4.

- a. Use Simulink to demonstrate RLS identification of  $\Theta = [a \ b]^T$  for  $y_k = ay_{k-1} + bu_{k-1} + w_k$ , where  $w_k$  is a normal white noise process.  
*Hint:* If you use Simulink, the block 'Matlab function' might be useful. Also type 'help persistent.' at the Matlab prompt.
- b. Comment on the choice of input signal and how it affects the result.
- c. Assume  $a$  is time varying, how can the method be modified to identify  $a$ ? How do you use information about typical speed of change of  $a$  to tune the algorithm?