

# Lecture 7: Anti-windup and friction compensation

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- ▶ Compensation for saturations (anti-windup)
- ▶ Friction models
- ▶ Friction compensation

## Material

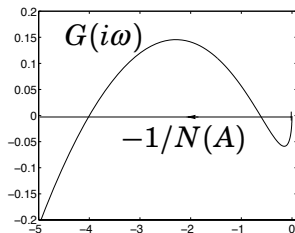
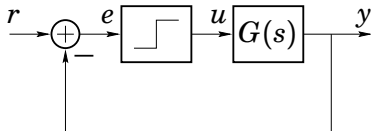
- ▶ Lecture slides

# Course Outline

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|--------------|---|
| Lecture 1-3  | Modelling and basic phenomena<br>(linearization, phase plane, limit cycles) |
| Lecture 2-6  | Analysis methods<br>(Lyapunov, circle criterion, describing functions)      |
| Lecture 7-8  | Common nonlinearities<br>(Saturation, friction, backlash, quantization)     |
| Lecture 9-13 | Design methods<br>(Lyapunov methods, Backstepping, Optimal control)         |
| Lecture 14   | Summary   |

## Last lecture: Stable periodic solution

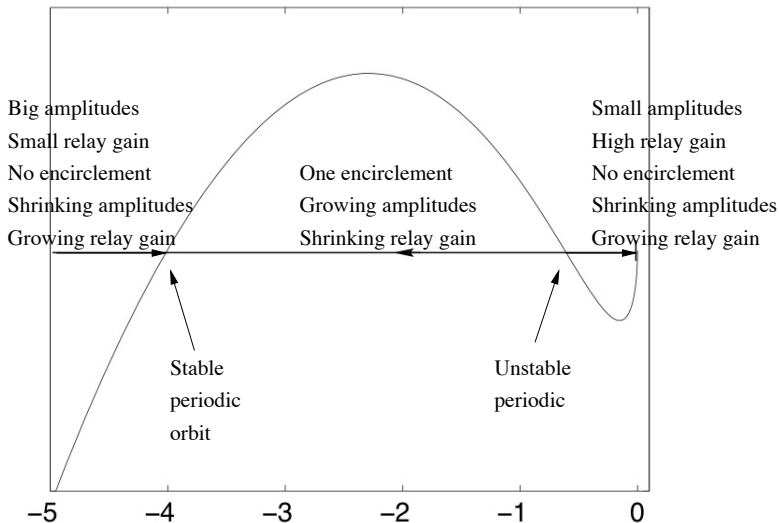


$$G(s) = \frac{(s+10)^2}{(s+1)^3} \quad \text{with feedback} \quad u = -\text{sgn } y$$

gives one stable and one unstable limit cycle. The left most intersection corresponds to the stable one.

# Periodic Solutions in Relay System

The relay gain  $N(A)$  is higher for small  $A$ :



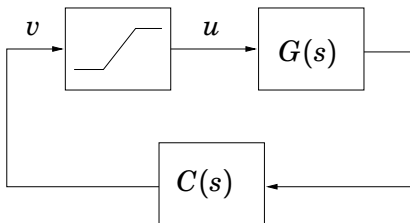
# Today's Goal

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- ▶ *To be able to design and analyze antiwindup schemes for*
  - ▶ *PID*
  - ▶ *state-space systems*
  - ▶ *and Kalman filters (observers)*
- ▶ *To understand common models of friction*
- ▶ *To design and analyze friction compensation schemes*

## Windup – The Problem

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The feedback path is broken when  $u$  saturates

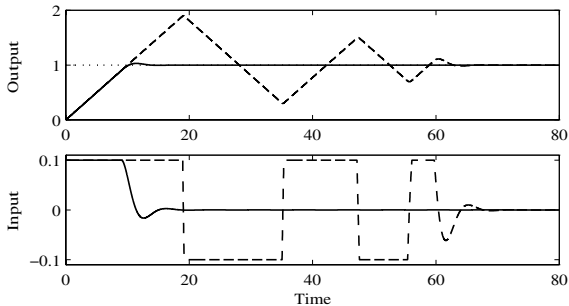
The controller  $C(s)$  is a dynamic system

Problems when controller is unstable (or stable but not AS)

Example: I-part in PID-controller

# Example-Windup in PID Controller

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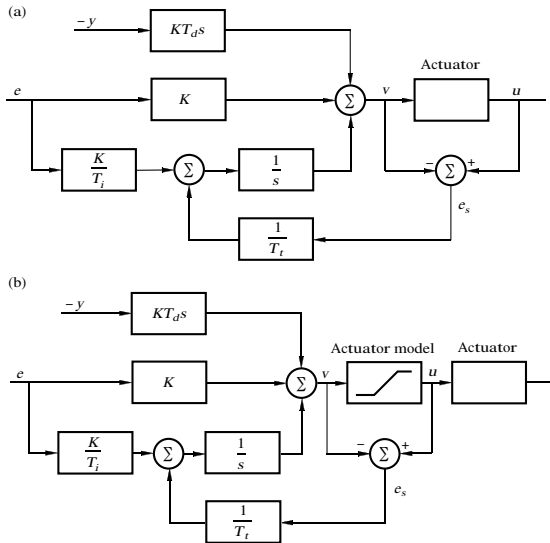


Dashed line: ordinary PID-controller

Solid line: PID-controller with anti-windup

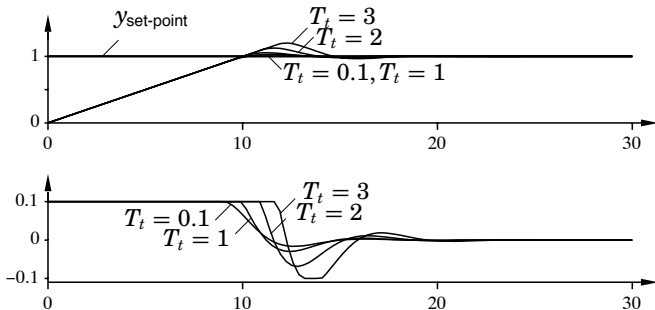
# Anti-windup for PID-Controller (“Tracking”)

Anti-windup (a) with actuator output available and (b) without





## Choice of Tracking Time $T_t$



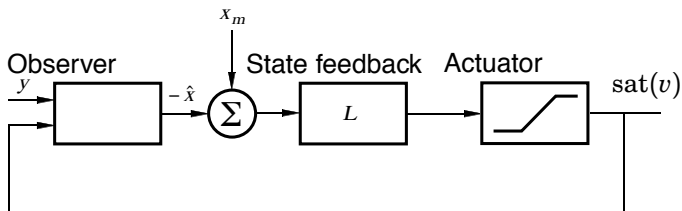
With very small  $T_t$  (large gain  $1/T_t$ ), spurious errors can saturate the output, which leads to accidental reset of the integrator. Too large  $T_t$  gives too slow reaction (little effect).

The tracking time  $T_t$  is the design parameter of the anti-windup.

Common choices:  $T_t = T_i$  or  $T_t = \sqrt{T_i T_d}$ .

# State feedback with Observer

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$$\dot{\hat{x}} = A\hat{x} + B \text{sat}(v) + K(y - C\hat{x})$$

$$v = L(x_m - \hat{x})$$

$\hat{x}$  is estimate of process state,  $x_m$  desired (model) state.  
Need model of saturation if  $\text{sat}(v)$  is not measurable

# Antiwindup – General State-Space Controller

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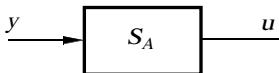
State-space controller:

$$\dot{x}_c(t) = Fx_c(t) + Gy(t)$$

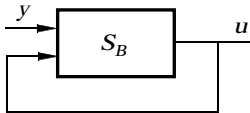
$$u(t) = Cx_c(t) + Dy(t)$$

Windup possible if  $F$  is unstable and  $u$  saturates.

(a)



(b)



Idea:

Rewrite representation of control law from (a) to (b) such that:

(a) and (b) have same input-output relation

(b) behaves better when feedback loop is broken, if  $S_B$  stable

# Antiwindup – General State-Space Controller

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Mimic the observer-based controller:

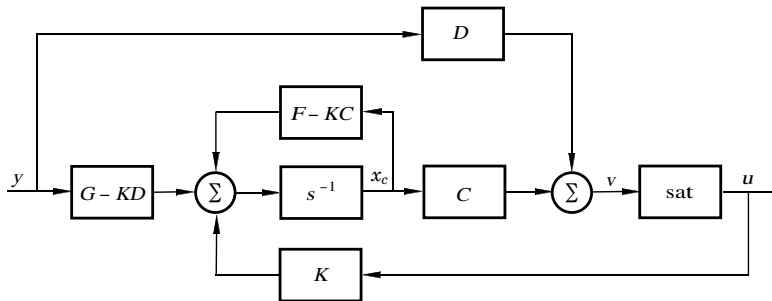
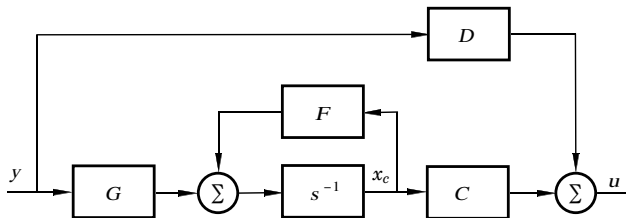
$$\begin{aligned}\dot{x}_c &= Fx_c + Gy + K \underbrace{(u - Cx_c - Dy)}_{=0} \\ &= (F - KC)x_c + (G - KD)y + Ku \\ &= F_0x_c + G_0y + Ku\end{aligned}$$

Design so that  $F_0 = F - KC$  has desired stable eigenvalues

Then use controller

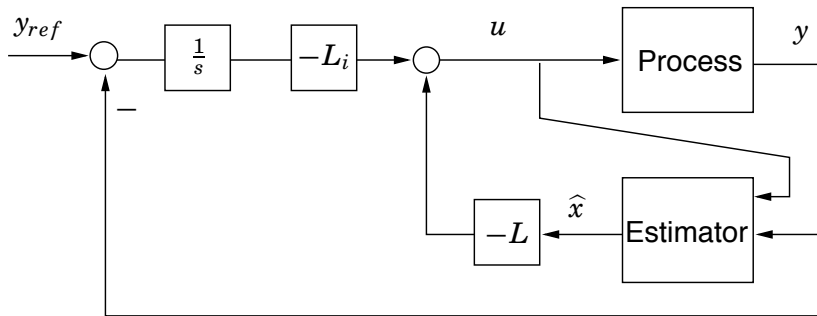
$$\begin{aligned}\dot{x}_c &= F_0x_c + G_0y + Ku \\ u &= \text{sat}(Cx_c + Dy)\end{aligned}$$

## State-space controller without and with anti-windup:



## 5 Minute Exercise

*How would you do antiwindup for the following state-feedback controller with observer and integral action ?*



# Saturation

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Optimal control theory (later)

## **Multi-loop Anti-windup (Cascaded systems):**

Difficult problem, several suggested solutions

Turn off integrator in outer loop when inner loop saturates

# Friction

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Almost is present almost everywhere

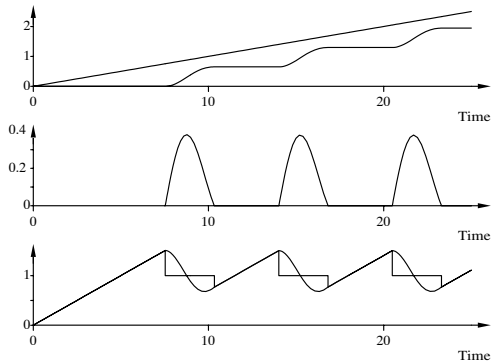
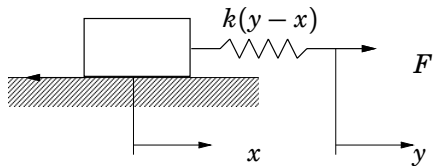
- ▶ Often bad
  - ▶ Friction in valves and mechanical constructions
- ▶ Sometimes good
  - ▶ Friction in brakes
- ▶ Sometimes too small
  - ▶ Earthquakes

Problems

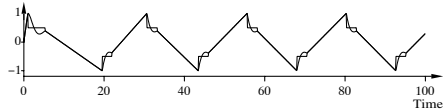
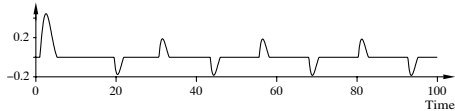
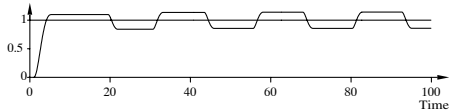
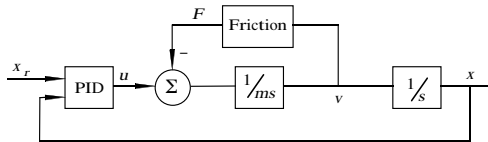
- ▶ How to model friction
- ▶ How to compensate for friction



# Stick-slip Motion



# Position Control of Servo with Friction – Hunting

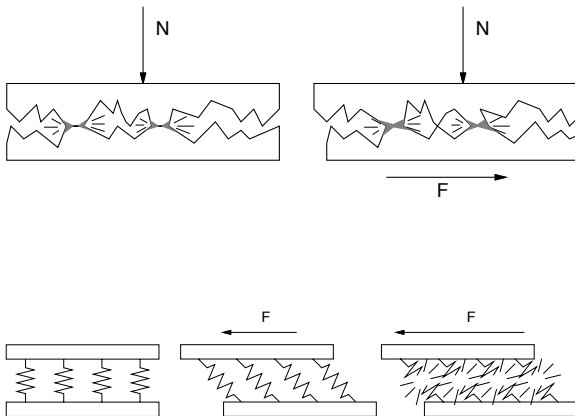


## 3 Minute Exercise

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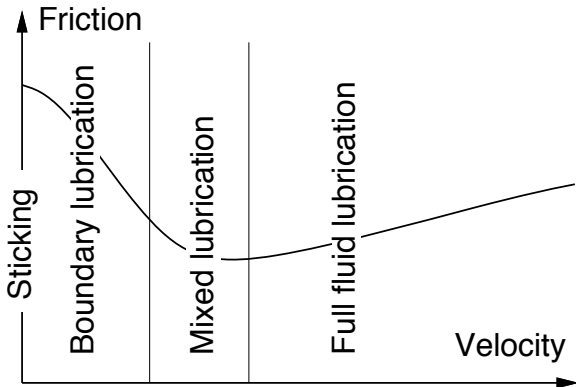
*What are the signals in the previous plots? What model of friction has been used in the simulation?*

# Friction



# Lubrication Regimes

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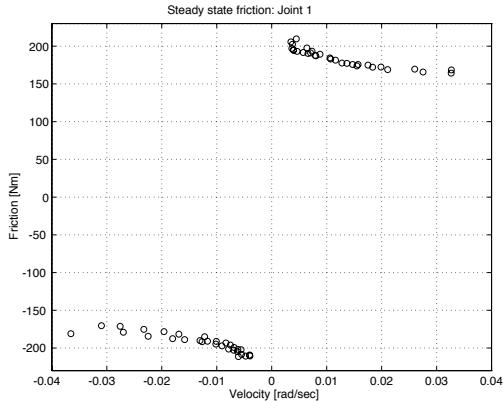


Hard to get good model at  $v = 0$

# Stribeck Effect

For low velocity: friction increases with decreasing velocity

Stribeck (1902)



# Frictional Lag

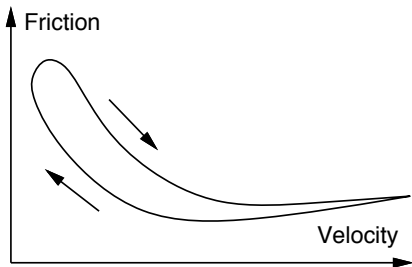
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Dynamics are important also outside sticking regime

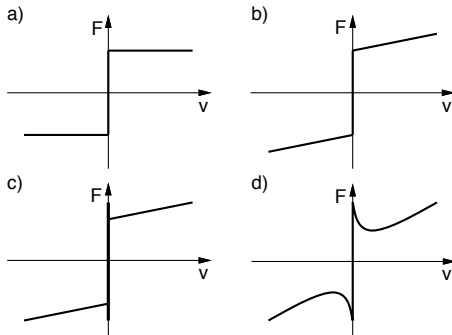
Hess and Soom (1990)

Experiment with unidirectional motion  $v(t) = v_0 + a \sin(\omega t)$

Hysteresis effect!



# Classical Friction Models



$$c) \quad F(t) = \begin{cases} F_c \operatorname{sign} v(t) + F_v v(t) & v(t) \neq 0 \\ \max(\min(F_e(t), F_s), -F_s) & v(t) = 0 \end{cases}$$

$F_e(t)$  = external applied force ,  $F_c, F_v, F_s$  constants



# Advanced Friction Models

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See PhD-thesis by Henrik Olsson

- ▶ Karnopp model
- ▶ Armstrong's seven parameter model
- ▶ Dahl model
- ▶ Bristle model
- ▶ Reset integrator model
- ▶ Bliman and Sorine
- ▶ Wit-Olsson-Åström

## Demands on a model

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To be useful for control the model should be

- ▶ sufficiently accurate,
- ▶ suitable for simulation,
- ▶ simple, few parameters to determine.
- ▶ physical interpretations, insight

Pick the simplest model that does the job! If no stiction occurs the  $v = 0$ -models are not needed.

# Friction Compensation

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- ▶ Lubrication
- ▶ Integral action (beware!)
- ▶ Dither
- ▶ Non-model based control
- ▶ Model based friction compensation
- ▶ Adaptive friction compensation

# Integral Action

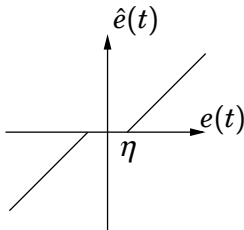
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- The integral action compensates for any external disturbance
- Good if friction force changes slowly ( $v \approx \text{constant}$ ).
- To get fast action when friction changes one must use much integral action (small  $T_i$ )
- Gives phase lag, may cause stability problems etc

## Deadzone - Modified Integral Action

Modify integral part to  $I = \frac{K}{T_i} \int_0^t \hat{e}(t) d\tau$

$$\text{where input to integrator } \hat{e} = \begin{cases} e(t) - \eta & e(t) > \eta \\ 0 & |e(t)| < \eta \\ e(t) + \eta & e(t) < -\eta \end{cases}$$

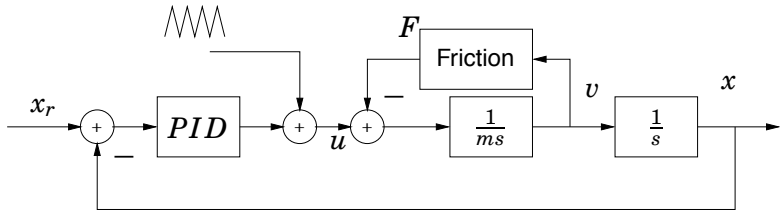


**Advantage:** Avoid that small static error introduces limit cycle

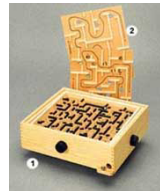
**Disadvantage:** Must accept small error (will not go to zero)

# Mechanical Vibrator–Dither

Avoids sticking at  $v = 0$  where there usually is high friction by adding high-frequency mechanical vibration (dither)

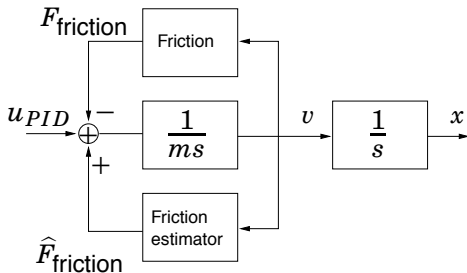


Cf., mechanical maze puzzle  
(labyrinthspel)



# Adaptive Friction Compensation

Coulomb Friction  $F = a \operatorname{sgn}(v)$



Assumption:  $v$  measurable.

Friction estimator:

$$\dot{z} = k u_{PID} \operatorname{sgn}(v)$$

$$\hat{a} = z - km|v|$$

$$\hat{F}_{friction} = \hat{a} \operatorname{sgn}(v)$$

Result:  $e = a - \hat{a} \rightarrow 0$  as  $t \rightarrow \infty$ ,

since

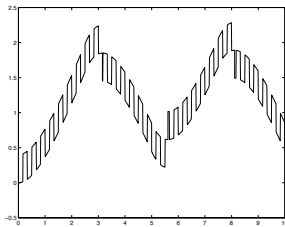
$$\begin{aligned}\frac{de}{dt} &= -\frac{d\hat{a}}{dt} = -\frac{dz}{dt} + km\frac{d}{dt}|v| \\ &= -ku_{PID} \operatorname{sgn}(v) + km\dot{v} \operatorname{sgn}(v) \\ &= -k \operatorname{sgn}(v)(u_{PID} - m\hat{v}) \\ &= -k \operatorname{sgn}(v)(F - \hat{F}) \\ &= -k(a - \hat{a}) \\ &= -ke\end{aligned}$$

Remark: Careful with  $\frac{d}{dt}|v|$  at  $v = 0$ .



## The Knocker

Combines Coulomb compensation and square wave dither



Tore Hägglund, Innovation Cup winner + patent 1997

# Example—Friction Compensation

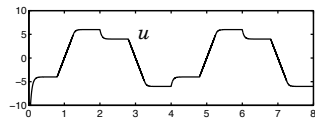
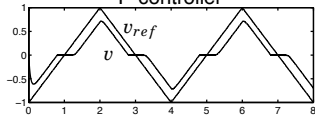
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Velocity control with

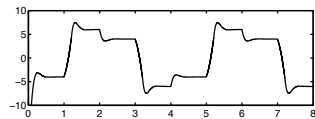
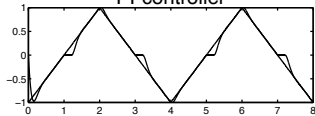
- a) P-controller
- b) PI-controller
- c) P + Coulomb estimation

# Results

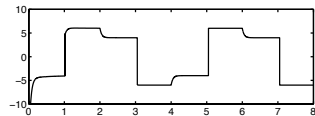
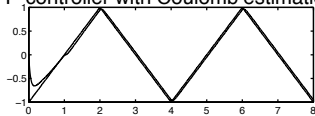
P-controller



PI-controller



P-controller with Coulomb estimation



## Next Lecture

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- ▶ Backlash
- ▶ Quantization