Lund University

Department of
AUTOMATIC CONTROL

Automatic Control, Basic Course (FRT010)

2105-12-17

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem.

Grade 3: 12 points
4: 17 points
5: 22 points

Accepted aid

Mathematical collections of formulae (e.g. TEFYMA), 'Collections of formulae in automatic control', and calculators that are not programmed in advance.

Results

The graded exam will be displayed on December 19, 08-10 am in building 4, room 303. Thereafter, exams will be archived at the Automatic Control department in Lund.

1. A certain system is described by the following differential equation:

$$\ddot{y}(t) + 3\dot{y}(t) + 12y(t) + 2\ddot{u}(t) - 5\dot{u}(t) = -2u(t).$$

a. Write the transfer function from the input u to the output y. (1 p)

b. If u is a unit step, what is the stationary value of y? (1 p)

c. What is the order of the system? (0.5 p)

d. Is the system linear or nonlinear? Motivate your answer. (0.5 p)

2. Consider the nonlinear system

$$\dot{x}_1 = -x_1 + x_2^2,$$
$$\dot{x}_2 = x_1 x_2 - u^3.$$

a. Find all stationary points of the system corresponding to $u^0 = 5$. (1 p)

b. Linearize the system around (one of) the stationary points found in the previous step. If you did not solve the previous problem, use the point $({x_1}^0, {x_2}^0, u^0) = (3, 9, 3)$. (2 p)

c. Find the poles of the linearized system and comment upon its stability properties. (2 p)

3. You are given a physical process with transfer function

$$G_p(s) = \frac{2}{s+3}.$$

Design a PI-controller so that all poles of the closed-loop system are -3. (2 p)

4. You are given a system

$$\begin{cases} \dot{x} &= \begin{bmatrix} 2 & 1 \\ 0 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 1 \end{bmatrix} x. \end{cases}$$

a. Is the system controllable? (1 p)

b. Design a state feedback controller

$$u = -Lx + l_r r,$$

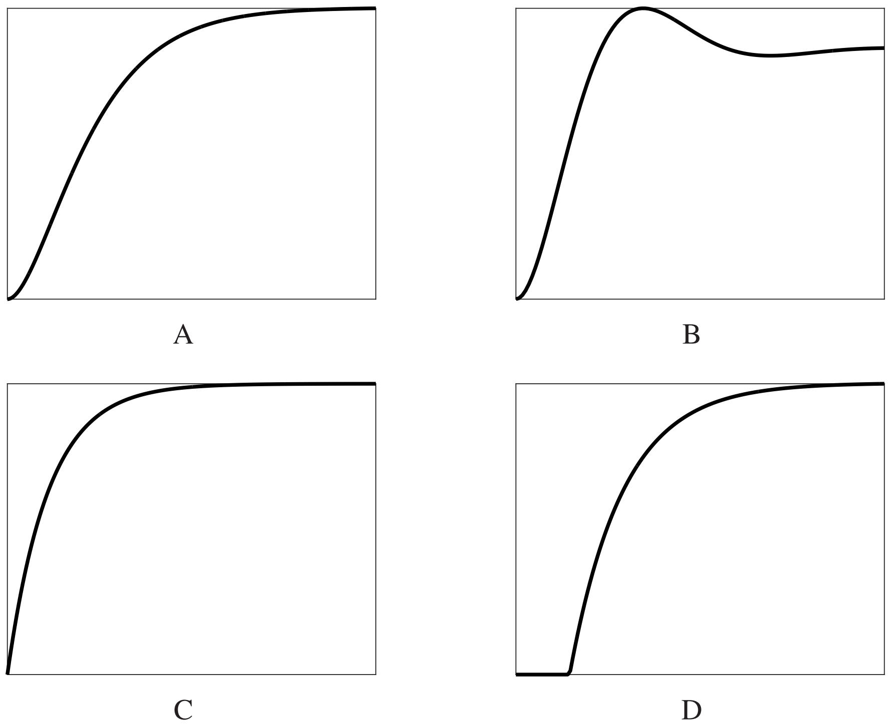
such that the resulting closed-loop system has its poles in $s = -4$ and $s = -5$. Determine l_r so that y = r in stationarity. (2 p)

c. Is the system observable? (1 p)

d. Suppose that measures of the states are not available. Design a Kalman filter to estimate the value of the states. Place the poles of the Kalman filter in -10. (2 p)

5. Pair the following transfer functions and step responses. Do not forget to motivate your answers. (2 p)

$$P_1(s) = \frac{1}{s+1}, \quad P_2(s) = \frac{1}{s+1}e^{-s}, \quad P_3(s) = \frac{1}{(s+1)^2}, \quad P_4(s) = \frac{1}{s^2+s+1}.$$



6. Are there situations where feed forward is preferential to feedback control? If so, give an example. (1 p)

7. You are faced with designing a P controller $C(s) = K$ for a process with dynamics

$$P(s) = \frac{2}{s(s+1)}.$$

You would like to maximize K, in order to minimize the stationary error due to load disturbances. However, due to model uncertainty, you need to maintain a 30° phase margin. What value of K do these design criteria result in? (3 p)

8. You are introduced to a control system with open-loop transfer function

$$G_o(s) = \frac{10}{s(s+1)}.$$

Performance is satisfactory in terms of robustness. However, your employer would like to make the control loop faster. Propose a solution that doubles the cross-over frequency, while maintaining the current phase margin. (3 p)