LUND
UNIVERSITY

Department of
AUTOMATIC CONTROL

Exam FRTF01 - Physiological Models and Computation

January 08 2018, 8-13

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem. Preliminary grades:

Grade 3: 12–16.5 points
4: 17 –21.5 points
5: 22 –25 points

Accepted aid

Lecture slides, any books (without relevant exercises with solutions), including the course textbooks. Standard mathematical tables and Collection of common results and formulae in control. Calculator.

Results

The result of the exam will be posted in LADOK no later than February 6. Information on when the corrected exam papers will be shown, will be given on the course homepage.

1.

a. Given the ion data in Tab. 1, calculate the equilibrium potentials of each of the three ions; Ion_1, Ion_2 and Ion_3, at room temperature, 25°C, by means of the Nernst equation.

Table 1: Ion concentrations and valence charge

Ion	**Inner conc. [μM]**	**External conc. [μM]**	**Valence charge**
Ion_1	25	150	+2
Ion_2	130	8.0	+1
Ion_3	6.2	10	-1

Use the following values for the constants, $R = 8.31447$ [J/mol·K] - thermodynamic gas constant, $F = 9.648534 \cdot 10^4$ [C/mol] - Faraday constant.

(1 p)

b. The temperature is lowered. What happens to the equilibrium potential of Ion_1, Ion_2 and Ion_3 (do they increase or decrease)? Motivate your answer.

(1 p)

2.

a. A patient is administered a drug through infusion into the blood. The current concentration of the drug in the patients blood is C_0 [mg/dl]. The concentration is governed by the kinetics $\dot{C} = -0.8C$ [mg/(dl· h)]. At what time is the concentration of the drug down to 40 % of the initial value? (1 p)

b. The doctors are tired of having to keep administering the drug to keep it close to the desired value C_{opt}. They ask you to decide which constant dose in [mg] to inject the patient with to keep the drug concentration at the level C_{opt} at steady state. The volume of the blood compartment is $V = 50$ [dl]. (2 p)

3. Consider the set of differential equations below

$$\begin{aligned}\dot{a} &= -k_{ab}a + k_{ba}b + k_{ca}c \\ \dot{b} &= k_{ab}a - k_{ba}b - k_{bc}b \\ \dot{c} &= k_{bc}b - k_{ca}c \\ y &= c/V_c\end{aligned}$$

where a, b and c are in [mg]. The compartments have distribution volumes V_a, V_b and V_c, respectively. The signal y is a measurement.

a. Draw a compartment model of the system. (1 p)

b. Write the system on state space form, that is $\dot{x} = Ax$, $y = Cx$. (1 p)

c. What can you say about the sum $a(t) + b(t) + c(t)$ given that $a(0) + b(0) + c(0) = p$, where p is some positive constant. (1 p)

4. Your colleague Doctor Dorian needs help in deciding the property of two inventions that use feedback on a specific physiological system.

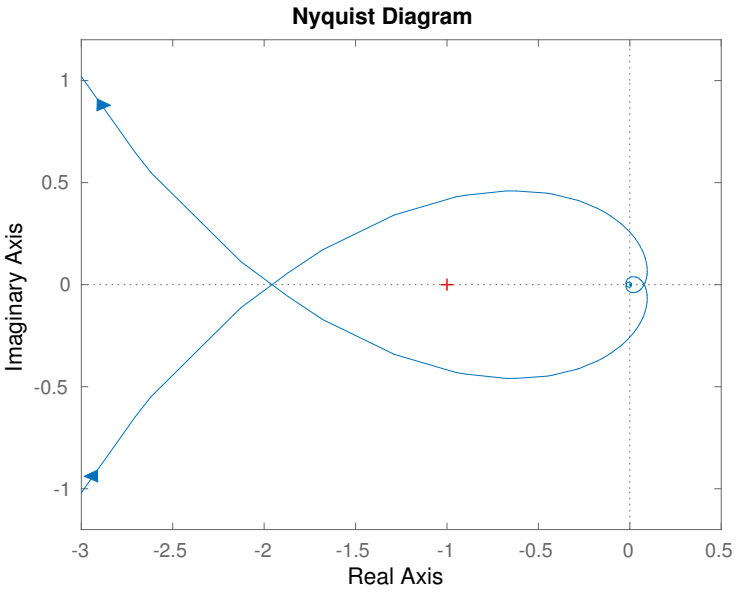


Figure 1: Nyquist plot of the system for problem 4a

a. The first invention utilizes simple output feedback, $u = K(r - y)$ where K is a scalar. Doctor Dorian has managed to find the Nyquist plot of the open-loop system, which can be seen in Fig 1 and tells you that the open-loop is stable. Determine the largest K such that the closed-loop system is stable. (1 p)

b. Draw a block diagram of the closed loop in the previous sub problem. (0.5 p)

c. The second invention is described by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

With

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Doctor Dorian suggests you use state feedback, $u = -Lx + l_r r$. Find the transfer function from r to y in terms of the matrices A, B and C. (1 p)

d. Design your controller so that the closed loop has its poles in $-1, -1$. (1.5 p)

5. The interaction between predators y, and pray x, can be described by the Lotka-Volterra equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha x - \beta xy$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \delta xy - \gamma y.$$

a. Find the stationary points (x^0, y^0) of the system. (1 p)

b. Linearize the system around a stationary point with positive (non-zero) x^0 and y^0. (2 p)

c. Determine if the linearized system is stable, marginally stable or unstable for $\alpha, \beta, \gamma, \delta > 0$. (1 p)

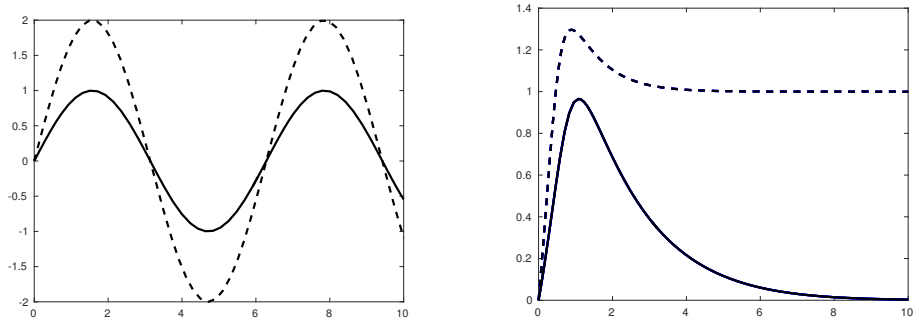
6.

a. Let the transfer function from u to y be given by $G(s) = 2/(s+3)$. Determine $y(t)$ when $u(t) = \sin(3\omega t)$. (1 p)

b. For the same system, determine $y(t)$ when $u(t) = \theta(t)$ (that is, u is a step). (1 p)

c. For any linear system with zero initial condition, what can you say about the relationship between two outputs $y_1(t)$ and $y_2(t)$ with inputs $u_1(t)$ and $u_2(t)$, respectively, when $2u_1(t) = u_2(t)$. (1 p)

d. In Figure 2 the step responses are plotted for two different step sizes. Which of the two systems can **not** be linear?



(a) System A (b) System B

Figure 2: Step response with amplitude 1 (solid) and amplitude 2 (dashed).

(1 p)

7. Match the step responses in Figure 3 with the transfer functions below:

$$\textbf{i)}\ \frac{1}{s+1} \qquad \textbf{ii)}\ \frac{0.5}{s+1}$$
$$\textbf{iii)}\ \frac{1}{s^2-0.1s+1} \qquad \textbf{iv)}\ \frac{e^{-s}}{1+s}$$

(Motivation is required for any points, including the last pair!) (2 p)

8. The data in Tab. 2 describes the substrate concentration, $[S]$, and reaction rate, V, of a chemical process. It is an enzymatic reaction following the Michaelis-Menten relationship, i.e,

$$V = \frac{V_{max}[S]}{K_m + [S]}.$$

Use the least-squares method to estimate the parameters V_{max} and K_m. (3 p)

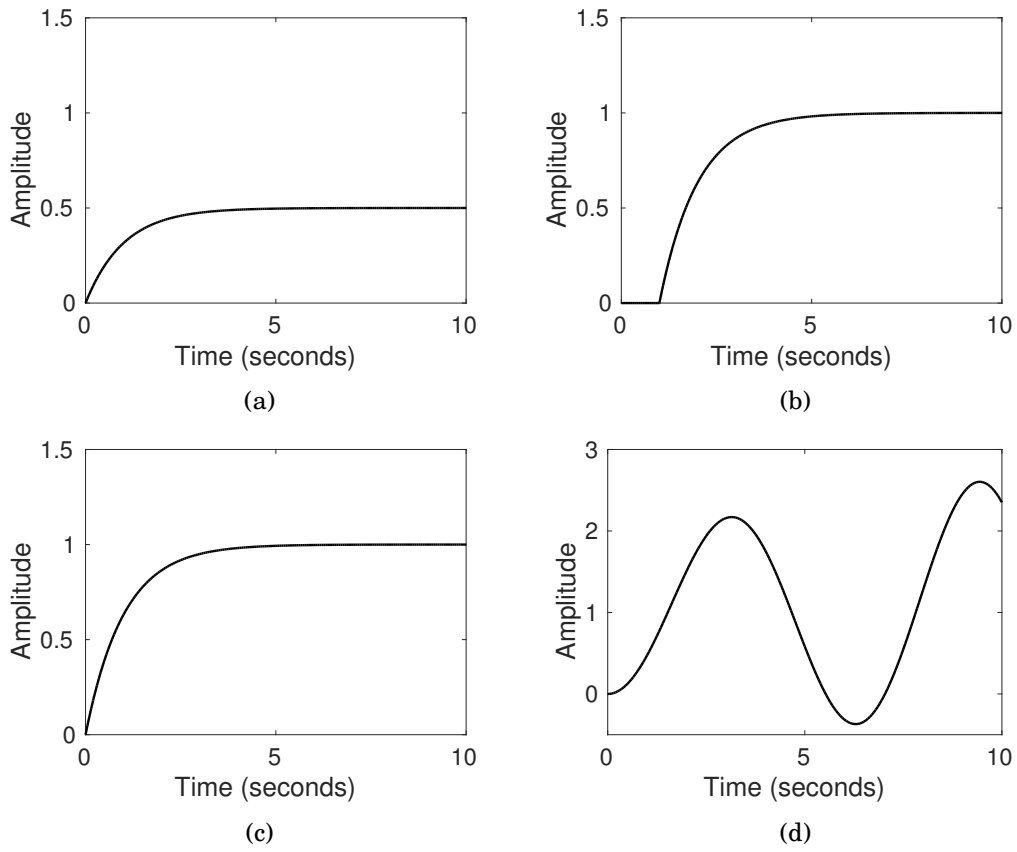


Figure 3: Step responses for problem 7.

Table 2: Drug data for problem 1

Substrate concentration $[S]$ [units]	Reaction rate V [units/days]
0.5	0.12
1	0.27
2	0.33
4	0.58

Good Luck!