LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

FRT130 Control Theory
Hand-in Assignment 2

To be submitted before March 25, 2015 at 17.00

Information About the Hand-in Assignments

The course FRT130 Control Theory contains two mandatory hand-in assignments. The exercises range from investigations of theoretical concepts to simulation exercises using numerical tools, such as MATLAB.

The intention of the exercises is that you should apply your knowledge from complex and linear analysis in order to solve the tasks given. Some of the exercises have more of a discussion character, and require that you actively use the available literature.

Hand-in Assignment 1 is performed in groups of two or three students, while Hand-in Assignment 2 is performed individually. Hand-in Assignment 2 should be submitted as a written report.

Report

The written report is an important part of the work, and the hand-in assignments are evaluated both with respect to content and layout. The report should be written such that engineering students on your own level can follow the solutions.

Apply the guidelines for writing reports that you have obtained in previous courses. Some hints:

- The report can be written with paper and pencil or using appropriate computer software. Write complete sentences.
- If possible, start by describing the problem to be solved. Also, explain the notation and variables that you introduce.
- Explain the different steps of your solution and provide logical motivations for them. Specify the theorems or results that you utilize (Pythagoras Theorem, Cauchy's Integral Formula, . . .).
- Please make sure that you verify that all questions in the exercises are answered in your solutions.
- Attach figures and plots if it contributes to the interpretation of the solutions. Also attach relevant code or scripts if you employ numerical tools for simulations and computations.

- It is allowed to discuss the exercises with other students on a general level, but each student should submit an individual report.

Grading

In the course FRT130 Control Theory, the grades are Pass and Fail. In order to pass the course, both hand-in assignments need to be approved. We use a scale 3, 2, 1, R when evaluating Hand-in Assignment 2. The scales correspond approximately to:

3: A good solution with appropriate presentation. The solution needs not to be absolutely correct.

2: An acceptable solution and presentation. The solution may have minor limitations.

1: A slightly incorrect solution or limitations in the presentation of the solution. Does not need to be re-submitted.

R: Revise the solution and submit a new version within one week. In some cases, the revised report will be followed by an oral discussion regarding the exercises.

1. Consider a process that resembles the spring-mass-damper system investigated in Laboratory Exercise 3 in the Basic Course in Control:

$$\begin{aligned} \frac{d^2x_1}{dt^2} &= (x_2 - x_1) - 0.1\frac{dx_1}{dt} + u, \qquad\qquad x(0) = x_0 \\ \frac{d^2x_2}{dt^2} &= (x_1 - x_2) - 0.1\frac{dx_2}{dt}, \\ y &= x_1 \end{aligned}$$

a. Use the command `place` in MATLAB in order to compute a state-feedback controller based on estimated states,

$$u = -L\hat{x} + l_r r,$$

that gives the following poles of the closed-loop system:

$$(s^2 + s + 3/2)(s^2 + s + 1),$$

and an observer

$$\dot{\hat{x}} = (A - KC)\hat{x} + Bu + Ky \qquad \hat{x}(0) = 0,$$

with the pole placement:

$$(s^2 + s + 3/2)(s^2 + 2\zeta_o\omega_o s + \omega_o^2),$$

where $\zeta_o = 0.5$ and $\omega_o = 1$. The static gain should be 1.

b. Determine the transfer function of the process on the format $Y(s) = P(s)U(s)$. Also, compute the transfer function of the controller according to the expression

$$U(s) = -G_{uy}(s)Y(s) + G_{ur}(s)R(s).$$

The transfer functions should be expressed in the variables A, B, C, L, and K.

c. Simulate the step response of the closed-loop system ($r(t) = 1$, $t \geq 0$)

$$\begin{aligned} \dot{x} &= Ax - BL\hat{x} + Bl_r r, & x(0) &= x_0 \\ \dot{\hat{x}} &= KCx + (A - KC - BL)\hat{x} + Bl_r r, & \hat{x}(0) &= 0, \end{aligned}$$

for different values of ω_o (for example, 0.75, 2, 10) in the cases when $x_0 = 0$ and $x_0 \neq 0$ (for example, $x_0 = [0\ 1]^T$). It might be helpful to introduce the matrices

```
A_e = [A,-B*L; K*C,(A-K*C-B*L)]
B_e = [B*l_r; B*l_r]
```

In MATLAB, the function `lsim` can be used. Alternatively, the simulation environment SIMULINK can be used for simulating the system. Explain the differences in the obtained simulation results. In particular, investigate the estimation error $\tilde{x} = x - \hat{x}$.

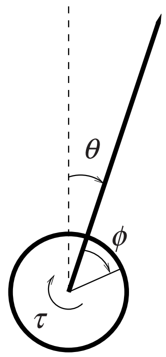

Figure 1 A schematic picture of a segway robot.

d. The loop-transfer function for the compensated system is given by $L(s) = P(s)G_{uy}(s)$. Plot the Bode diagram and determine the amplitude and phase margins for the case $\omega_o = 1$ (use `margin` in MATLAB).

Also, plot the magnitude of the sensitivity function, $|S(i\omega)|$, and determine its maximum value M_s. What does this value of M_s mean for the robustness of the controlled system?
Use `bodemag` in order to compute the magnitude of the sensitivity function. M_s can also be computed as $||S(s)||_\infty$. (Extra: Explain why these two approaches are equivalent).

2. Consider the segway robot illustrated in Figure 1. The pendulum angle relative to the vertical plane is θ and the wheel angle is ϕ. The applied torque is denoted τ. The equations of motion can then be modeled by Euler-Lagrange mechanics, which result in the model

$$\begin{pmatrix} J_p & m_p l r \cos\theta \\ m_p l r \cos\theta & J_w + m_w r^2 + m_p r^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\phi} \end{pmatrix} + \begin{pmatrix} -m_p g l \sin\theta \\ -m_p l r \dot{\theta}^2 \sin\theta \end{pmatrix} = \begin{pmatrix} -\tau \\ \tau \end{pmatrix}$$

where $J_p = 0.16$ kgm^2 is the moment of inertia of the body with respect to the pivot, $m_p = 2.9$ kg is the mass of the pendulum, $l = 0.17$ m is the distance from the pivot from the body center of mass, $m_w = 0.5$ kg, $r = 0.05$ m, $J_w = 0.0005$ kgm^2 are the mass, radius and moment of inertia of the wheel assembly, respectively.

a. Linearize the dynamics around $(\theta, \dot{\theta}, \phi, \dot{\phi}) = (0, 0, 0, 0)$ and write the state equation on the form

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -c & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ -b \\ 0 \\ d \end{pmatrix} \tau$$

Determine the expressions for a, b, c, d. What is the state x?

b. Determine the transfer function from τ to ϕ.

c. Determine the zeros and poles of the system, when τ is the input and ϕ is the output.

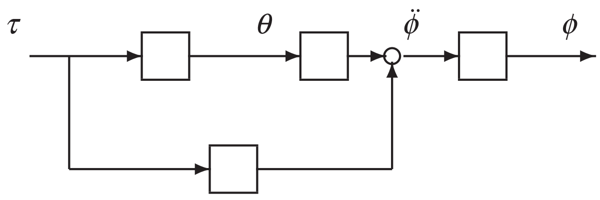

Figure 2 Block diagram for the linearized segway dynamics.

d. Is the system observable using the angle θ as measurement signal? If not, determine the unobservable subspace.

e. Determine conditions on the parameters a, b, c, d for the system *not* to be controllable. Relate the conditions to the expression for the transfer function derived in Ex. b). Which states can be reached in these cases?

f. Determine the transfer functions in the empty boxes in Figure 2.

g. Can you stabilize the upright position of the pendulum using a feedback law of the form $\tau = k_1\theta + k_2\dot{\theta}$? Suggest suitable values of the parameters k_1 and k_2 that result in an acceptable sensitivity of the closed-loop system. In particular, plot the Bode diagram for the sensitivity function for your choice of parameters.

3.

a. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u, \quad x(0) = 0.$$

Determine the piece-wise constant input signal u,

$$u(t) = \begin{cases} u_1, & 0 \le t \le 1/2 \\ u_2, & 1/2 < t \le 1 \end{cases}$$

that minimizes

$$\underset{u}{\text{minimize}} \int_0^1 u(t)^2 \mathrm{d}t$$

$$\text{subject to: } x(1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

b. What happens in the case that

$$u(t) = \begin{cases} u_1, & 0 \le t \le 1/3 \\ u_2, & 1/3 < t \le 2/3 \\ u_3, & 2/3 < t \le 1 \end{cases}$$

c. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u, \quad x(0) = 0,$$

and redo the computations in Ex. a). Explain the result.

Matlab Hints

Practical hints to using MATLAB can be found in the document *A Matlab Tutorial*, available on the course homepage. Particularly useful commands are:

`ss`	Create a state-space model
`tf`	Create a transfer function
`bode`	Plot Bode diagram
`bodemag`	Plot Bode magnitude
`place`	Pole placement
`obsv`	Observability matrix
`ctrb`	Controllability matrix
`zero`	Compute zeros
`pole`	Compute poles
`lsim`	Simulate a state-space system