

Lektion 1

- Kursinnehåll - kursprogram
- Det praktiska - boken - idag **sid 71-101**
- AKbakgrund - frekvenskurvor **AK 27-39**
- Mattebakgrund - Spannes Blixtkurs
- Laplacetransform **AK 17**
- Koppling till tillståndsbeskrivning **AK 18**
Betoning av transienter och inverkan av initialtillstånd

1

AKbakgrund – Frekvenskurvor

- Frekvenskurvor
 $u(t) = \sin \omega t$, $y(t) = A(\omega) \sin(\omega t + \varphi(\omega))$
 $A(\omega) = |G(i\omega)|$, $\varphi(\omega) = \arg G(i\omega)$
- Representation av $G(s)$ och $G(i\omega)$
- Nyquistdiagram - komplexa talet $G(i\omega)$
- Bodediagram - $|G(i\omega)|$ och $\arg G(i\omega)$
 $G = G_1 G_2 G_3 G_4 \dots$

2

Mattebakgrund - Spannes Blixtkurs

- $\int_C f(z) dz$, $C: \{z(t), t \in [a, b]\}$, $\int_a^b f(z) \frac{dz}{dt} dt$, **återför på flerdim**
- $f(z) = \frac{1}{z-a}$, $C: \{z(t) = a + re^{it}, t \in [0, 2\pi]\}$, **enda exemplet**
- $f(z)$ analytisk, sluten kurva, Cauchys integralsats, olika vägar lika, **deformation av integrationsväg**
- $\int_C \frac{f(z)}{z-a} dz = f(a) 2\pi i$, **Cauchys integralformel**
- $\{z_k\}_1^n$ poler innanför C till $f(z)$, $\int_C = \int_{C_1} + \dots + \int_{C_n}$,
 $\text{Res}_{z=z_i} f(z) = \frac{1}{2\pi i} \int_{C_i} f(z) dz$, **residuekalkyl**

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Laplacetransform

- Dubbelsidig/enkelsidig - Kausala system
- Definitionsremsa. Olika för olika signaler?
- Överföringsfunktion. Oberoende av insignal?
- Enkelsidig plus analytisk fortsättning - Ex
- Klarar även instabila system

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Laplacetransform - definition - konvergens

Dubbelsidig: Betrakta tidsfunktioner $f(t)$, $-\infty < t < \infty$

$$F(s) = (\mathcal{L}_I f)(s) = \int_{-\infty}^{\infty} e^{-st} f(t) dt$$

Konvergerar i remsa $\Omega: \alpha < \text{Re } s < \beta$, $F(s)$ analytisk i Ω .

$$f(t)e^{-\alpha t} \rightarrow 0, \quad t \rightarrow \infty, \quad \text{och} \quad f(t)e^{-\beta t} \rightarrow 0, \quad t \rightarrow -\infty.$$

Tex $\alpha < 0$ och $\beta > 0$ kräver exp. konvergens $t \rightarrow \infty$, och $t \rightarrow -\infty$.

Enkelsidig: Betrakta tidsfunktioner $f(t)$, $0 \leq t < \infty$

$$F(s) = (\mathcal{L}_I f)(s) = \int_0^{\infty} e^{-st} f(t) dt$$

Konvergerar i halvplan $\Omega: \alpha < \text{Re } s$, $F(s)$ analytisk i Ω .

$$f(t)e^{-\alpha t} \rightarrow 0, \quad t \rightarrow \infty, \quad \text{dvs } \alpha > 0 \text{ tillåter } f(t) \rightarrow \infty, \quad t \rightarrow \infty.$$

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Laplacetransform - kausalitet

Observera att

$$\mathcal{L}_I \frac{df}{dt} = sF(s) - f(0)$$

Viktfunktion

$$y(t) = \int_0^t h(t-s)u(s)ds = \int_0^t h(\tau)u(t-\tau)d\tau$$

$$h(\tau), \quad 0 \leq \tau < \infty$$

$$G(s) = (\mathcal{L}h)(s)$$

$$Y(s) = G(s)U(s)$$

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Lapacetransform - exempel

$$f(t) = e^{2t}, t \geq 0, \quad F = \mathcal{L}\{f\}, \quad F(s) = \lim_{T \rightarrow \infty} \int_0^T e^{2t} e^{-st} dt$$

$$F(s) = \lim_{T \rightarrow \infty} \left[\frac{1}{2-s} e^{(2-s)t} \right]_0^T = \frac{1}{2-s} \lim_{T \rightarrow \infty} \{ e^{(2-s)T} - 1 \}$$

$$\lim_{T \rightarrow \infty} e^{(2-s)T} = 0, \quad \text{Re } s > 2$$

Alltså

$$F(s) = \frac{1}{s-2}, \quad \text{Re } s > 2$$

Utvidga defområdet med analytisk fortsättning till $\mathbb{C} - \{s = 2\}$, enda möjliga funktionen.

Transienter och initialtillstånd

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad x(0) = x_0$$

Lapacetransformering ger

$$\begin{aligned} sX(s) - x_0 &= AX(s) + BU(s) \\ X(s) &= (sI - A)^{-1} (BU(s) + x_0) \\ Y &= CX + DU = \underbrace{[C(sI - A)^{-1}B + D]}_{G(s)} U(s) + C(sI - A)^{-1}x_0 \end{aligned}$$

Exempel: Sinus in och inverkan av initialtillstånd

$$\dot{x} = -x + u \quad x(0) = x_0 \quad u(t) = \sin t$$

ger efter Lapacetransformering

$$sX(s) - x(0) = -X(s) + U(s) \quad U(s) = \frac{1}{s^2 + 1}$$

Här kan X lösas ut:

$$\begin{aligned} X(s) &= \frac{1}{s+1} (U(s) + x_0) = \frac{1}{s+1} \left(\frac{1}{s^2+1} + x_0 \right) \\ &= \frac{0.5 - 0.5s}{s^2+1} + \frac{0.5 + x_0}{s+1} \end{aligned}$$

Inverstransformering ger

$$x(t) = \frac{1}{2} \sin t - \frac{1}{2} \cos t + \left(x_0 + \frac{1}{2} \right) e^{-t}$$

The Power of Feedback

Feedback has some amazing properties, it can

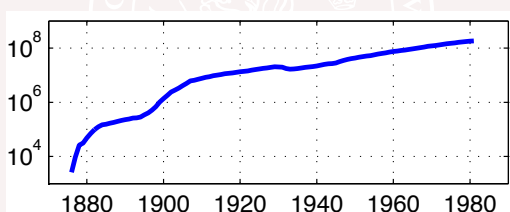
- make a system insensitive to disturbances,
- make good systems from bad components,
- follow command signals
- stabilize an unstable system,
- create desired behavior, for example linear behavior from nonlinear components.

The major drawbacks are that

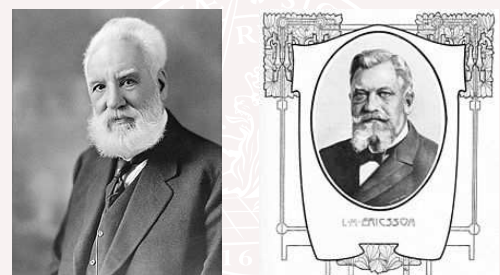
- feedback can cause instabilities
- sensor noise is fed into the system

Introduction

- Driving force: Telecommunications a rapidly growing industry
- Alexander Graham Bell 1847-1922
- Patent 1876
- No patent in Sweden!
- Lars Magnus Ericsson 1846-1926



Bell and Ericsson



The AT&T Research Laboratories

Bennet 2 p 70–71: *The AT&T Company formed an industrial research laboratory as part of its strategy of controlling all American telecommunications, summarized by its then President, Theodore Vail, as 'One Policy, One System, Universal Service'. To implement the strategy the company needed to control the rate and direction of technical change by obtaining, or preventing others from obtaining, key patents; and it also needed to avoid being broken up under the Sherman Antitrust Act. The research laboratories played a major part in ensuring that the company kept control of the technology and the patent rights.*

The transistor was also invented at Bell Labs!

The Repeater Problem

- The electro mechanical repeater
- 6mm wire 280 kg/km
- 1911 East coast to Denver
- 1914 First transcontinental New York San Francisco
- 1915 Improved transcontinental three vacuum tube repeaters, two repeaters added in 1916 and two more in 1918.

System	Date	Ch pair	Loss db 3000mi	Repeat 3000mi
1st TC	1914	1	60	3-6
2nd TC	1923	1-4	150-400	6-20
Open W	1938	16	1000	40
Cable	1936	12	12.000	200
Coaxial	1941	480	30.000	600

Bode - Feedback The History of an Idea

Bode, H. W. Feedback - The History of an idea. Proc. Symp Active Networks and Feedback Systems. Polytechnic Institute of Brooklyn, 1960. in Bellman and Kalaba. Selected papers on mathematical trends in control theory. Dover 1964.

Most of you with hi-fi systems are no doubt proud of your audio amplifiers, but I doubt whether many of you would care to listen to the sound after the signal had gone in succession through several dozen or several hundred even of your fine amplifiers. There is a 'tyranny of numbers, as my reliability friends say, which makes it necessary for the individual components of the system to become qualitatively better as the system as a whole becomes quantitatively more ambitious.

Distortion in Cascaded Amplifiers

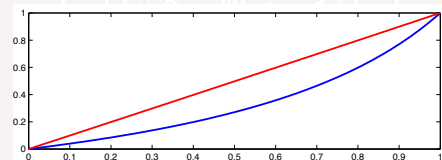
Compositions of functions

$$f_n = f \circ f \dots \circ f$$

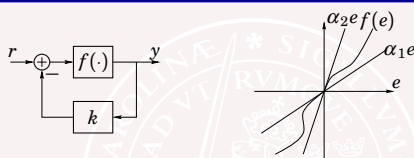
Define distortion as

$$d_n = 2 \frac{\max f'_n(x) - \min f'_n(x)}{\max f'_n(x) + \min f'_n(x)}$$

Example $f(x) = \frac{x + ax^2}{1 + a}$, $a = 0.01$, $\Rightarrow d_1 = 0.01, d_{100} = 0.74$



Linearization Through High Gain



$$\alpha_1 e \leq f(e) \leq \alpha_2 e \quad \text{gives} \quad \frac{\alpha_1}{1 + \alpha_1 k} r \leq y \leq \frac{\alpha_2}{1 + \alpha_2 k} r$$

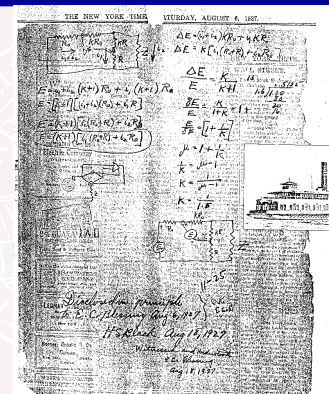
so if $\alpha_1 k, \alpha_2 k \gg 1$ then

$$y \approx \frac{1}{k} r$$

regardless of the nonlinearity. (Easier to design $k < 1$ with high accuracy.)

Black's Original

- Patents
- Disclosures
- Engineering notebooks



Nine years in the Patent Office

Although the invention had been submitted to the U.S. Patent Office on August 8, 1928, more than nine years would elapse before the patent was issued on December 21, 1937 (No. 2 102 671). One reason for the delay was that the concept was so contrary to established beliefs that the Patent Office initially did not believe it would work. The Office cited technical papers, for example, that maintained the output could not be connected back to the input unless the loop gain was less than one, whereas mine was between 40 and 50 dB. In England, our patent application was treated in the same manner as one for a perpetual-motion machine. Burgess was eventually able to overcome all these objections by submitting evidence that 70 amplifiers were working successfully ...

Mervin Kelly on Black IEEE Lamme Medal 1957

Although many of Harold's inventions have made great impact, that of the negative feedback amplifier is indeed the most outstanding. It easily ranks coordinate with De Forest's invention of the audion as one of the two inventions of broadest scope and significance in electronics and communications of the past 50 years....it is no exaggeration to say that without Black's invention, the present long-distance telephone and television networks which cover our entire country and the transoceanic telephone cables would not exist. The application of Black's principle of negative feedback has not been limited to telecommunications. Many of the industrial and military amplifiers would not be possible except for its use.

References

- ❶ S. Bennett A history of Control Engineering 1930-1955 Peter Peregrinus, IEE 1993.
- ❷ H. S. Black. Inventing the negative feedback amplifier. IEEE Spectrum. December 1977, 55–60.
- ❸ H. W. Bode, Relations between attenuation and phase in feedback amplifier design, BSTJ **13** (1940), 421–454.
- ❹ H. W. Bode, Feedback - The History of an idea. Proc. Symp Active Networks and Feedback Systems. Polytechnic Institute of Brooklyn, 1960. In Bellman and Kalaba. Selected papers on mathematical trends in control theory. Dover 1964.
- ❺ H. W. Bode Network Analysis and Feedback Amplifier Design. Van Nostrand, Princeton 1945.