LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

Exam in FRT110 Systems Engineering and FRTN25 Process Control

June 5, 2015, 14:00–19:00

Points and grades

All answers must include a clear motivation. Answers may be given in English or Swedish. The total number of points is 20 for Systems Engineering and 25 for Process Control. The maximum number of points is specified for each subproblem. Preliminary grading scales:

Systems Engineering:		Process Control:	
Grade 3:	10 points	Grade 3:	12 points
4:	14 points	4:	17 points
5:	17 points	5:	21 points

Acceptable Aid

Authorized *Formelsamling i reglerteknik / Collection of Formulae*. Standard mathematical tables like TEFYMA. Pocket calculator.

Results

The solutions are posted on the course home page, and the results will be transferred to LADOK. Date and location for display of the corrected exams will be posted on the course home page.

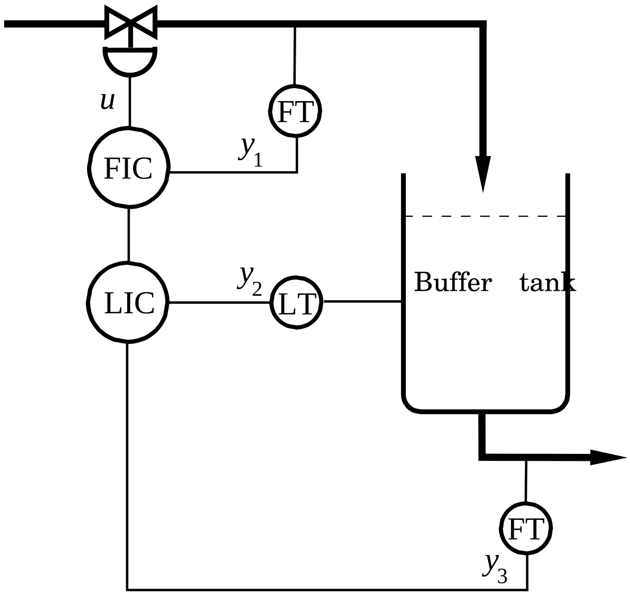

Figure 1 The level control system in Problem 1

Solutions to Exam in Systems Engineering/Process Control 2015-06-05

1. Figure 1 shows a P/I diagram for level control of a buffer tank. The process has one control signal, u, and three measurement signals, y_1, y_2, and y_3.

a. Two of the measurement signals are used for feedback—which? One of the measurement signals is used for feedforward—which? Explain! (1 p)

b. The current solution is deemed too expensive by your boss, and you have been ordered to remove two of the sensors (transmitters). Which sensors could you remove and still have a working system? Explain! (1 p)

Solution

a. y_1 and y_2 are used for feedback control in a cascaded structure. The master level controller (LIC) calculates the flow setpoint for the slave flow controller (FIC). y_3 measures the outflow, which can be seen as a disturbance. The level controller uses feedforward from this signal to compensate for the disturbance.

b. If y_2 is removed, there will be no feedback from the tank level, which is the quantity we are trying to control. Hence, y_2 should not be removed. Removing y_1 (and FIC) would disable the inner loop of the cascade controller, and removing y_3 would disable the feedforward. Both of these actions would decrease the control performance but still leave a working system.

2. Consider the nonlinear dynamical system

$$\begin{aligned}
\dot{x}_1 &= -8x_2^3 + \sqrt{u} \\
\dot{x}_2 &= x_1 \\
y &= x_2^2
\end{aligned}$$

where x_1 and x_2 are the state variables, u is the input and y is the output.

a. Determine the stationary state (x_1^0, x_2^0) and the stationary output y^0, when the input signal is constant, $u^0 = 1$. Then linearize the system around this stationary point. (2 p)

b. Determine the transfer function $G(s)$ that describes the relation between the input and the output of the linearized system. Determine the poles of the system and comment on its stability properties. (2 p)

Solution

a. In stationarity we have $\dot{x} = 0$, which yields $0 = -8(x_2^0)^3 + \sqrt{1} \leftrightarrow x_2^0 = 0.5$. Further, $x_1^0 = 0$. Finally, we have $y^0 = (x_2^0)^2 = 0.25$. Let

$$\begin{aligned} f_1(x_1, x_2, u) &= -8x_2^3 + \sqrt{u} \\ f_2(x_1, x_2, u) &= x_1 \\ g(x_1, x_2, u) &= x_2^2 \end{aligned}$$

The partial derivatives are

$$\begin{array}{lll} \dfrac{\partial f_1}{\partial x_1} = 0 & \dfrac{\partial f_1}{\partial x_2} = -24x_2^2 & \dfrac{\partial f_1}{\partial u} = \dfrac{1}{2\sqrt{u}} \\ \dfrac{\partial f_2}{\partial x_1} = 1 & \dfrac{\partial f_2}{\partial x_2} = 0 & \dfrac{\partial f_2}{\partial u} = 0 \\ \dfrac{\partial g}{\partial x_1} = 0 & \dfrac{\partial g}{\partial x_2} = 2x_2 & \dfrac{\partial g}{\partial u} = 0 \end{array}$$

Inserting the stationary values and introducing the variables $\Delta x = x - x^0, \Delta u = u - u^0$ and $\Delta y = y - y^0$, we get the linearized system

$$\begin{aligned} \dot{\Delta x} &= \begin{pmatrix} 0 & -6 \\ 1 & 0 \end{pmatrix} \Delta x + \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \Delta u \\ \Delta y &= \begin{pmatrix} 0 & 1 \end{pmatrix} \Delta x \end{aligned}$$

b. The transfer function is calculated as

$$G(s) = C(sI - A)^{-1}B = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & 6 \\ -1 & s \end{pmatrix}^{-1} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} = \frac{0.5}{s^2 + 6}$$

The two poles are located in $\pm\sqrt{6}\,i$. Hence, the system is marginally stable (i.e., stable but not asymptotically stable).

3. Figure 2 shows the pole-zero maps of four systems, and Figure 3 shows the step responses of the same systems, but not necessarily in the same order. The transfer function of each system can be written in the form

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

Match the pole-zero maps 1–4 with the corresponding step responses A–D. Motivate! (2 p)

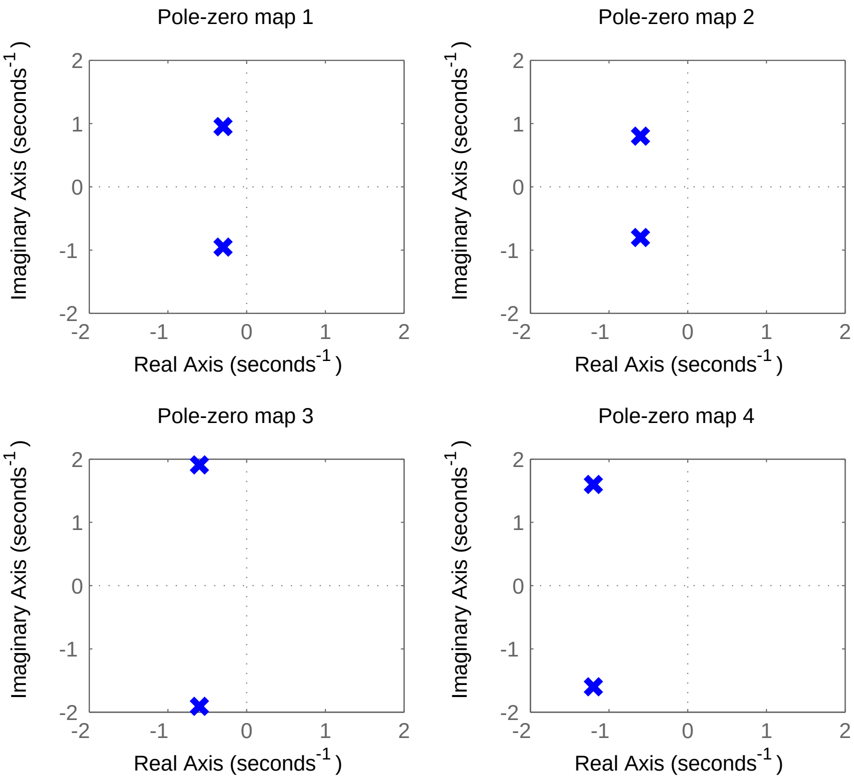

Figure 2 Pole-zero maps for Problem 3

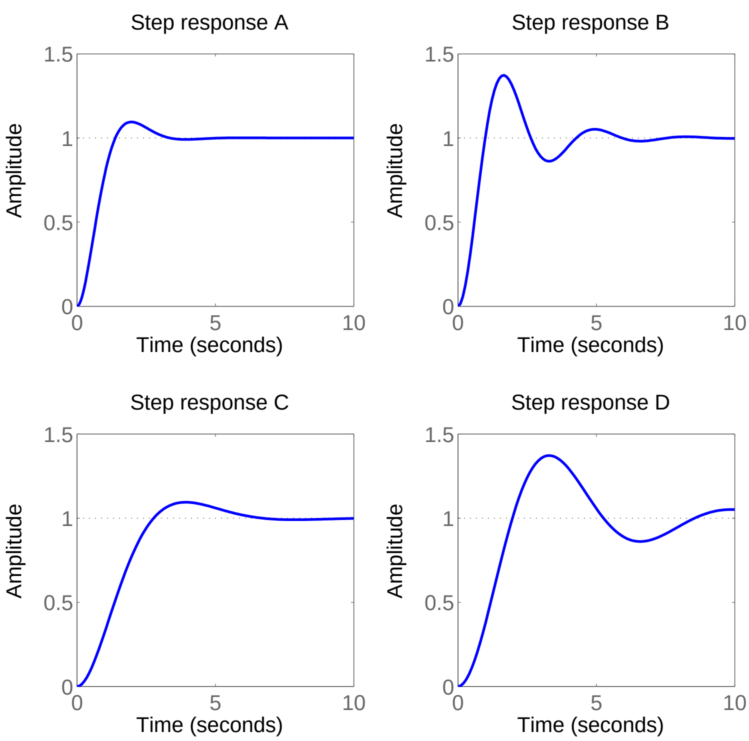

Figure 3 Step responses for Problem 3

Solution

Larger ω_0 gives faster step response, and poles further away from the origin. Larger ζ gives a more damped step response, and smaller angle between the poles and the negative real axis (see page 45 in the course book). Hence, the correct matching is 1–D, 2–C, 3–B, 4–A.

4. A process is described by the following differential equation, where u is the input and y is the output.

$$3\dot{y}(t) + y(t) - u(t - 0.1) = 0$$

a. Calculate the transfer function of the process. What are the static gain, the time constant, and the deadtime of the process? (2 p)

b. Calculate the impulse response of the process. (1 p)

Solution

a. Laplace transformation of the equation yields

$$3sY(s) + Y(s) - e^{-0.1s}U(s) = 0$$

Solve for $Y(s)$ to obtain

$$Y(s) = \frac{e^{-0.1s}}{3s+1}U(s)$$

Now, the transfer function can be identified as

$$G(s) = \frac{e^{-0.1s}}{3s+1}$$

- The system is asymptotically stable, so the static gain is given by $G(0) = 1$.
- The time constant is identified as $T = 3$.
- The deadtime is identified as $L = 0.1$.

b. We have $U(s) = 1$, and hence $Y(s) = \frac{e^{-0.1s}}{3s+1}$. Inverse Laplace transformation gives

$$y(t) = \begin{cases} 0, & t < 0.1 \\ \frac{1}{3}e^{-(t-0.1)/3}, & t \geq 0.1 \end{cases}$$

5. Consider an ideal PD controller

$$G_c(s) = K(1 + sT_d)$$

a. Calculate the gain of the controller for a sinusoidal input $e(t) = \sin \omega t$. (1 p)

b. What happens to the controller gain for large frequencies, and why is this a problem? How can the problem be handled in a practical implementation of the controller? (1 p)

Solution

a. The gain is calculated as

$$|G(i\omega)| = \left|K(1 + i\omega T_d)\right| = K\sqrt{1 + (\omega T_d)^2}$$

b. Very large input frequency gives a very large gain, and the controller is very sensitive to high-frequency noise. This can be coped with by using the following approximation of the derivative term, where the derivative gain is limited by N:

$$D(s) = \frac{sKT_d}{1 + sKT_d/N}$$

6. Consider the block diagram in Figure 4, where the transfer functions are assumed to be

$$G_1(s) = 3\left(1 + \frac{1}{5s}\right)$$
$$G_2(s) = \frac{2-s}{5s+1}$$

a. What type of controller is $G_1(s)$? Motivate! (1 p)

b. Calculate the transfer function $G_{yr}(s)$ from r to y. What are its poles and zeros? (2 p)

c. Determine, if possible, the static gain of $G_{yr}(s)$. Assuming a constant reference value r, what does this imply for the static error $e(\infty)$? (1 p)

Solution

a. This is a PI-controller, since it is in the form

$$G_1(s) = K\left(1 + \frac{1}{sT_i}\right)$$

The s in the denominator corresponds to time integration, and forms the integral part together with the control parameters.

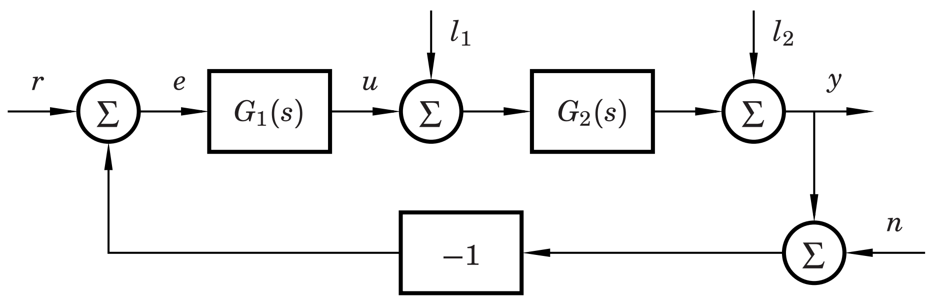

Figure 4 Block diagram in Problem 6 (and Problem 10).

b. Setting $l_1 = l_2 = n = 0$, we obtain

$$Y(s) = G_2(s)G_1(s)(R(s) - Y(s))$$
$$Y(s) = \underbrace{\frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)}}_{=G_{yr}(s)} R(s)$$

Inserting the given transfer functions, we calculate

$$G_{yr}(s) = \frac{-1.5(s+0.2)(s-2)}{(s+0.2)(s+3)} = \frac{-1.5(s-2)}{s+3}$$

The zero is hence located in $+2$, and the pole is located in -3.

(The process pole in -0.2 is cancelled by the controller zero in the same location—this is typical for Lambda tuning.)

c. Since the system is asymptotically stable (pole strictly in left half-plane), the static gain can be found as $G(0) = 1$. This implies that there will be no static error, since we will have $y(t) = r(t)$ in stationarity (again assuming $l_1 = l_2 = n = 0$).

7. The Nyquist plot of a linear system $G(s)$ is shown in Figure 5. You may assume that all poles of the system are located in the left half-plane.

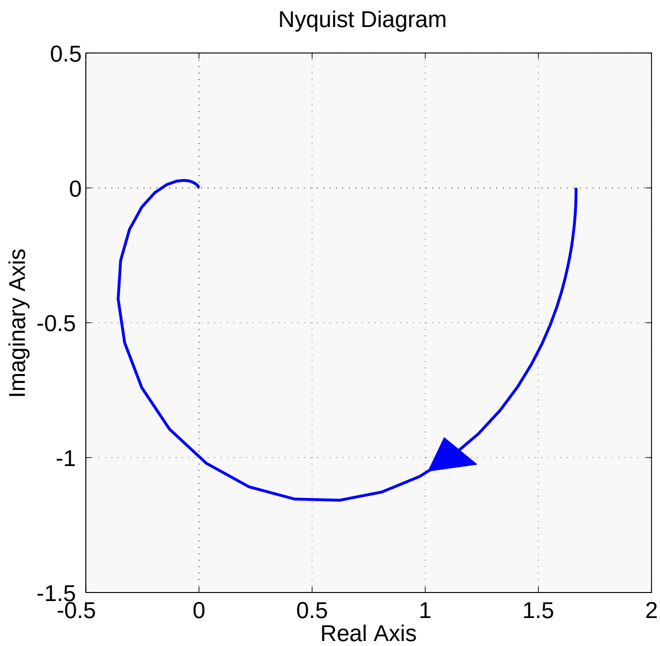

Figure 5 Nyquist plot of $G(s)$ in Problem 7.

a. Estimate the phase margin, the amplitude margin, and the static gain of the system. (1.5 p)

b. Let the system be feedback interconnected as shown in Figure 6 with $K \geq 0$. Let $d = 1$ be a constant disturbance. Calculate the stationary error $e(\infty)$ as a function of K. What is the smallest stationary error that may be achieved? (1.5 p)

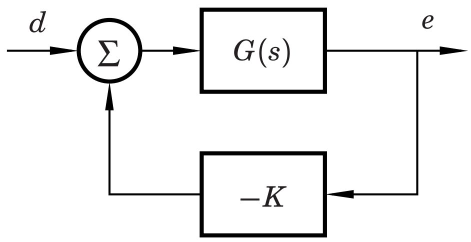

Figure 6 Feedback system in Problem 7.

Solution

a. The phase-margin is 90 deg as seen by the intersection of the unit-circle and the Nyquist curve. The Nyquist curve intersects the negative real axis at about -0.2 (the actual value is $-\frac{1}{6}$), which gives an amplitude margin of 5 (respectively 6).

The static gain is given by $G(0)$, which is about 1.6 (the actual value is $\frac{5}{3}$).

b. In the Laplace domain, we have

$$E(s) = G(s)(D(s) - KE(s)) \Leftrightarrow E(s) = \frac{G(s)}{1 + KG(s)} D(s)$$

With $D(s) = \frac{1}{s}$, we have

$$e(\infty) = \lim_{s \to 0} sE(s) = \frac{G(0)}{1 + KG(0)} = \frac{5/3}{1 + 5K/3}$$

The stationary error decreases with larger K, and for stability we need $K < 6$. Hence, the smallest stationary error that is possible is 0.15.

8. Only for FRTN25 Process Control. You have designed a sequential controller for a batch reactor and used GRAFCET to describe the logic, see Figure 7. The controller works as follows:

- The sequence starts when the user pushes the Start button (Start becomes true).
- The reactor is filled with reactant by the pump (action P). When the desired level is reached (L_2 becomes true), the pump is stopped.
- The heating of the reactor is now started (action Q). The heating stops when the temperature reaches the desired value (T becomes true).
- The reactor is now waiting for the reaction to finish. When the operator pushes the Empty button (Empty becomes true), the output valve is opened (action V). When the reactor is empty (L_1 becomes false), the valve is closed and the program returns to the initial step.

Modify the program so that the following new features are implemented:

- Filling and heating should be done in parallel. Heating may however not start until the low level sensor L_1 becomes true.
- It should be possible to abort the parallel filling and heating at any time by pressing the Empty button. (3 p)

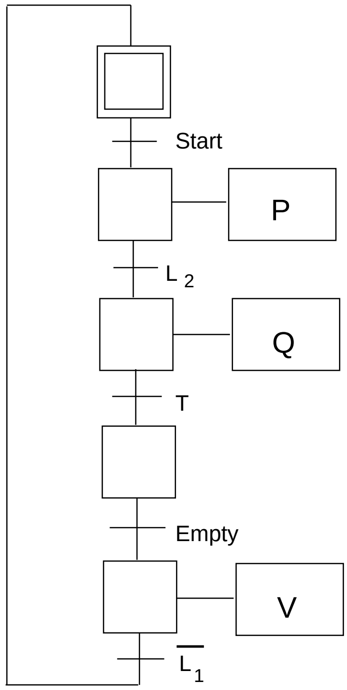

Figure 7 GRAFCET in Problem 8

Solution

One solution is shown in Figure 7. More elaborate solutions are also possible.

9. **Only for FRTN25 Process Control.** The relative gain array for a 4×4 model of a distillation column has been computed as follows:

$$\Lambda = \begin{pmatrix} 0.931 & 0.150 & 0.08 & -0.164 \\ -0.011 & -0.429 & 0.286 & 1.154 \\ -0.135 & 3.314 & -0.27 & -1.91 \\ 0.215 & -2.03 & 0.90 & 1.919 \end{pmatrix}$$

Using four simple controllers, which output signal $y_1 \ldots y_4$ should be fed back to what input signal $u_1 \ldots u_4$? Motivate! (1 p)

Solution

For each row in Λ (each corresponding to an output), one should find a column (each corresponding to an input) with a non-negative element close to one. In row 1, the first element is 0.931. In row 2, the fourth element is 1.154. In row 3, the only positive element is the second one, 3.314. In row 4, the third element is 0.90. A suitable pairing is hence y_1–u_1, y_2–u_4, y_3–u_2, y_4–u_3.

10. **Only for FRTN25 Process Control.** Again consider the block diagram in Figure 4, but now assume that $G_1(s)$ and $G_2(s)$ are arbitrary multivariable systems of matching dimensions. Calculate the multivariable transfer function from the reference signal vector r to the control signal vector u. (1 p)

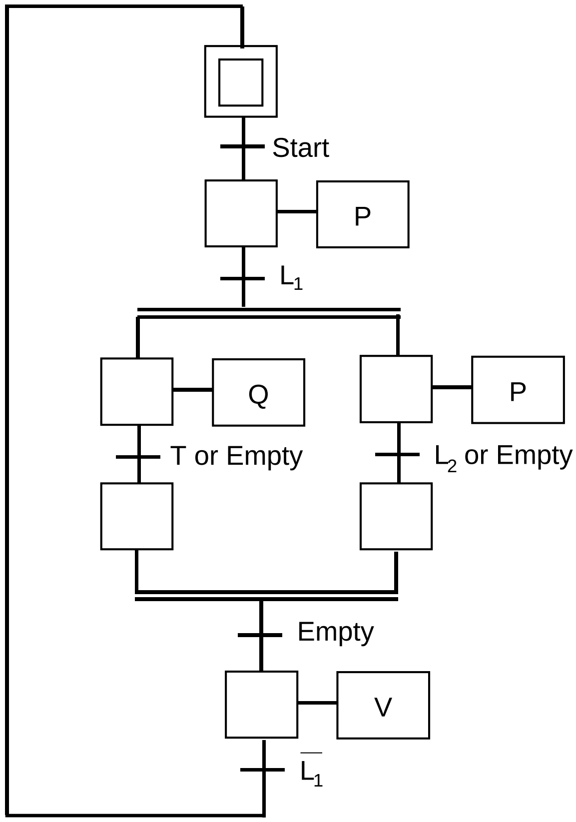

Figure 8 GRAFCET in Problem 8

Solution

The control signal vector is given by

$$U(s) = G_1(s)(R(s) - G_2(s)U(s))$$

Solving for $U(s)$ we obtain

$$\begin{aligned}(I + G_1(s)G_2(s))U(s) &= G_1(s)R(s)\\ U(s) &= (I + G_1(s)G_2(s))^{-1}G_1(s)R(s)\end{aligned}$$

The multivariable transfer function from r to u is hence given by

$$G_{ur}(s) = (I + G_1(s)G_2(s))^{-1}G_1(s)$$