LUNDS UNIVERSITET

Institutionen för
REGLERTEKNIK

FRT 041 System Identification

Final Exam October 29, 2014, 8am - 1pm

General Instructions

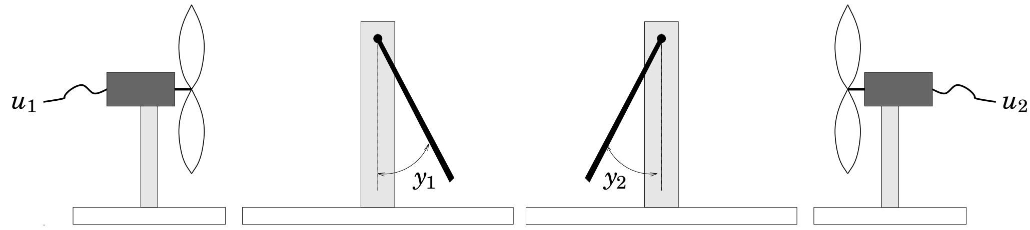
This is an open book exam. You may use any book you want, including the slides from the lecture, but no exercises, exams, or solution manuals are allowed. Solutions and answers to the problems should be well motivated. The exam consists of 9 problems. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits:

Grade 3:	12 – 16 points
Grade 4:	17 – 21 points
Grade 5:	22 – 25 points

Results

The results of the exam will be posted at the latest November 10, 2014 on the note board on the first floor of the M-building.

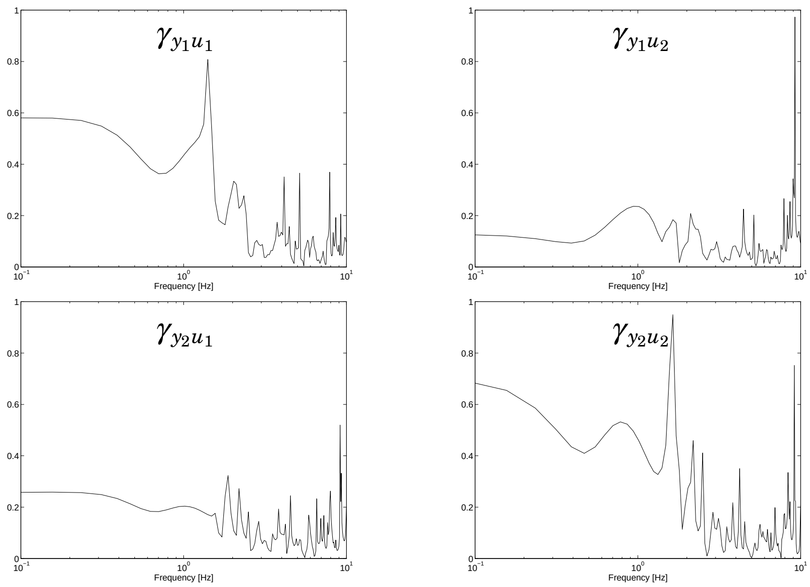
1. Consider the double fan and plate process below. The process has two inputs u_1 and u_2 and two outputs y_1 and y_2.



a. Two different PRBS signals are applied to the inputs and data are collected. The goal is to identify four transfer functions $H_{ij}(q)$, where $H_{ij}(q)$ is the transfer function from u_j to y_i, $i, j = 1, 2$. The coherence functions

$$\gamma_{ij}(\omega) = \frac{|S_{u_j y_i}(i\omega)|}{\sqrt{S_{u_j u_j}(i\omega) S_{y_i y_i}(i\omega)}}, \qquad i, j = 1, 2$$

are calculated and shown below. In what frequency range can we assume the estimated models to have good accuracy? Also, comment on the relative accuracy of the models, that is, can we assume that some transfer function estimates will have higher accuracy than others?



(2 p)

b. Assume we want to control the plate angles y_1 and y_2 with two PID controllers C_1 and C_2. There are two possibilities to connect the controllers: either we choose the coupling $u_1 = C_1 y_1$ and $u_2 = C_2 y_2$, or the coupling $u_1 = C_1 y_2$ and $u_2 = C_2 y_1$. Which one do you think will give best control performance? Motivate. (1 p)

2. Consider a nonlinear system of Hammerstein type. The linear and nonlinear parts are described as

$$y_k = b_1 x_{k-1} - a_1 y_{k-1} - a_2 y_{k-2},$$
$$x_k = g(u_k) = c_0 + c_1 u_k + c_2 u_k^2.$$

Formulate the parameter estimation of this model given that we have N samples of the input and the corresponding output. You can assume that static gain of the linear part is equal to one. (2 p)

3. Consider the model

$$y(t) = \theta + e(t),$$

where $e(t) \in \mathcal{N}(0, \sigma^2)$.

a. Using RLS method, find an update rule for θ. (1 p)

b. Find the least-squares estimate of θ and show how the same update rule can be obtained. (1 p)

4. The following model has been obtained from the estimation of an ARX model from input data u and output y.

$$\frac{Y(z)}{U(z)} = \frac{z-1}{(z-0.99)(z-0.8)}$$

a. Calculate the state-space model of the given system in the controllable canonical form. (1 p)

b. Given that the matrix T will transform the system from the controllable canonical form to the balanced realization, motivate whether it is advisable to reduce the system order. (2 p)

$$T^{-1} = \begin{bmatrix} -0.7363 & 22.3504 \\ 0.2638 & 22.3622 \end{bmatrix}, \quad T = \begin{bmatrix} -1.0001 & 0.9995 \\ 0.0118 & 0.0329 \end{bmatrix}$$

c. Perform model reduction according to the balanced realization method irrespective of the answer to **b**. Is the result equivalent to cancelling the pole at $z = 0.99$ with the zero at $z = 1$? (1 p)

5. Consider the system

$$\mathcal{S} : y_k = \varphi_k^T \theta + e_k$$

where v_k is white noise with the probability density function

$$f_e(e_k) = \alpha e^{-\beta |e_k|^p} \qquad p \in [1, \infty)$$

Measurements $y_1, \ldots, y_N$ and $u_1, \ldots, u_N$ are available. We are interested in obtaining a Maximum Likelihood estimate of parameters a and b from the available measurements.

a. Give the likelihood function as a function of parameter vector $\bar{\theta}$ and state the optimization problem that should be solved to obtain the Maximum Likelihood estimate of θ. (2 p)

b. How does the sensitivity of the ML estimate toward outliers change with p. (1 p)

6. Consider the model

$$y(t) = c\sin(\omega t + \phi) + e(t),$$

where c and ϕ are regarded as unknown constants while ω is known and $e(t) \in \mathcal{N}(0, \sigma^2)$. Given measurements $\{y_i\}_{i=1}^N$ derive an estimate of c and ϕ, for example using least squares estimation. (3 p)

7. Consider the true system,

$$y_k = u_{k-1} + u_{k-2} + e_k,$$

where e_k is white noise uncorrelated with all other noise terms. You try to fit the model,

$$y_k = bu_{k-1} + e_k,$$

where again e_k is uncorrelated with all other signals. Asymptotically ($N \to \infty$) find the best linear unbiased estimate of b if

a. $u_k = c$

b. $u_k = (-1)^k$

c. u_k white noise with variance σ^2 (3 p)

8. You are to perform system identification on a certain process operating under closed-loop conditions. The process output $\{y_k\}$ can be described by

$$y_k + ay_{k-1} \quad = \quad bu_{k-1} + e_k$$

where $\{e_k\}$ is a white noise process with $E\{e_k\} = 0$ and $E\{e_k^2\} = \sigma_e^2$. The input u_k is determined by the control law $u_k = K(r_k - y_k)$, where $\{r_k\}$ is also a white noise process, independent of $\{e_k\}$ and with $E\{r_k\} = 0$ and $E\{r_k^2\} = \sigma_r^2$. Use your method of choice to show that in the special case of $\sigma_r^2 = 0$, the estimation of the process model will fail. (3 p)

9. The impulse response coefficients (or Markov parameters) $\{h_k\}_{k=1}^\infty$ form the transfer function

$$H(z) = \sum_{k=1}^{\infty} h_k z^{-k}, \quad h_k = CA^{k-1}B$$

a. Show that a Hankel matrix of these coefficients can be factorised as

$$\begin{aligned}
\mathcal{H}_{r,s}^{(k)} &= \begin{pmatrix} h_{k+1} & h_{k+2} & \cdots & h_{k+s} \\ h_{k+2} & h_{k+3} & \cdots & h_{k+s+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{k+r} & h_{k+r+1} & \cdots & h_{k+r+s-1} \end{pmatrix} \\
&= \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{r-1} \end{pmatrix} A^k \begin{pmatrix} B & AB & \dots & A^{s-1}B \end{pmatrix}
\end{aligned}$$

(1 p)

b. How can this fact be exploited for system identification purposes? (1 p)