Robust Control 2018

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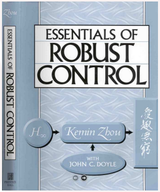
Robust Control, 9hp

- 7 Lectures, 7 exercises
- Examination: Exercises + Handins + Exam
- Book: Essentials of Robust Control, Zhou/Doyle
- Tools: Matlab (a Julia-initiative is encouraged)
- Schedule: See home page, details TBD now



Contents

Lecture 1 - today

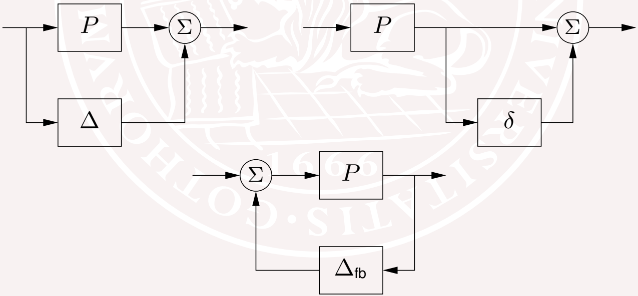
- Self-study, exercises, handins, challenges
- Robustness for scalar systems, GangOfFour
- Robustness for MIMO systems, singular values?
- Why H_∞ control ?
- How define 'closeness' of systems? The Vinnicombe metric

Robust Control - Introduction

- Process dynamics may change
- Feedback can deal with process variations
- How to characterize uncertainty

  Parameter variations, more general variations, unmodeled dynamics

- Additive Δ, multiplicative δ and feedback uncertainty Δ_{fb}



Non-robustness - reality strikes back

A wise man has said (and many agree with me):

"You often pay for lack of understanding in your control system design by non-robustness."

Unmodeled dynamics, too simplistic models, too aggressive control design, too high bandwidth, badly formulated optimization criterion, variations in parameters, lack of understanding of fundamental performance limitations . . .

Let's do PI control of a first order system together:

Example - PI control of 1st order system

Normalizing input, output and time variables let's assume

$$P(s) = \frac{1}{s+1}$$

Assume the pole at -1 actually corresponds here to fast dynamics in a measurement sensor.

Advice from expert:
"For this process you shouldn't aim to achieve a closed loop bandwidth as fast as the sensor dynamics, that would probably give large control signals and non-robustness. Aim a decade lower. It's good that you modeled the sensor dynamics. Try PI control and pole placement."

Example - PI control of 1st order system

Let's follow the advice

$$P(s) = \frac{1}{s+1}, \qquad C(s) = k_p + \frac{k_i}{s}$$

Pole-placement design

$$(s+1)(k_p s + k_i) = s^2 + 2\zeta_0\omega_0 s + \omega_0^2$$

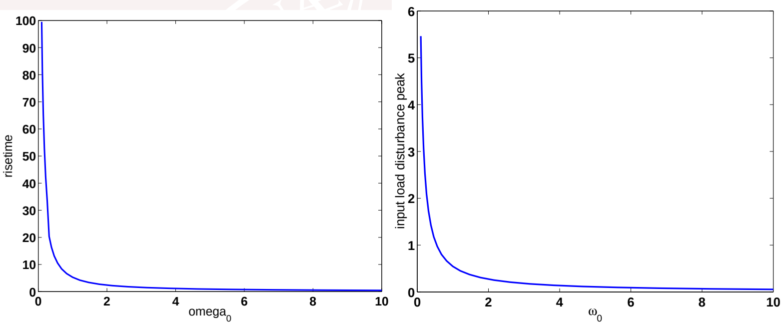
We get

$$\begin{aligned} k_p &= 2\zeta_0\omega_0 - 1 \\ k_i &= \omega_0^2 \end{aligned}$$

We expect a closed loop time bandwidth of 1/10th of the sensor bandwidth to be realistic.

Therefore we guess $\omega_0 = 0.1, \zeta_0 = 0.5$. But let's evaluate different ω_0

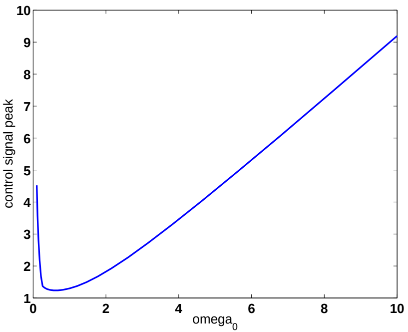
Result - Rise time vs ω_0



Trade-off looks as expected.

Let's check the control signal size also.

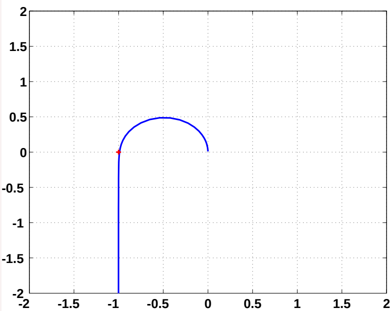
Step Response - input signal size

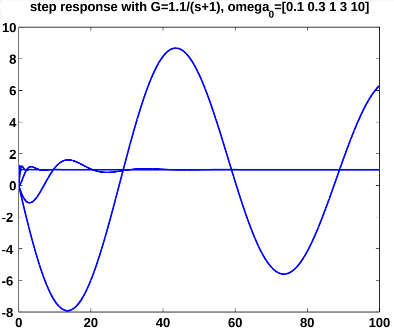


Hmm, the behavior when ω_0 is small is rather unexpected.

Let's check the Nyquist diagram for $\omega_0 = 0.1$ and do some simulations

Nyquist Diagram ω_0=0.1





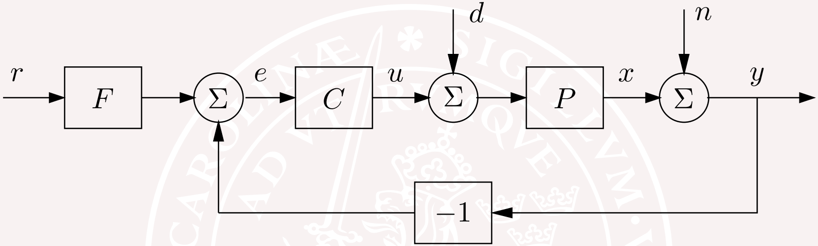
The design with $\omega_0 = 0.1$ has terrible robustness. The system becomes unstable with $\sim$ 10 % gain change

Reasonable design choices, but result is practically useless!

Was any of the expert advice bad? Choice of PI? The pole-placement?

(Handin 1a, explain and make a better design)

The Gangs of Four and Seven



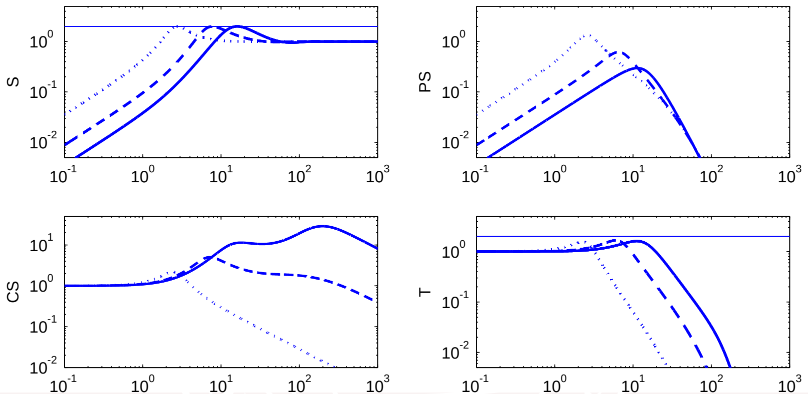
$$X = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$

$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$

$$E = -\frac{P}{1+PC}D - \frac{1}{1+PC}N + \frac{F}{1+PC}R$$

$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Gain Curves of the Gang of Four



Gain curves of the Gang of Four for a heat conduction process with I (dash-dotted), PI (dashed) and PID (full) controllers.

One plot gives a good overview of performance and robustness!

Analysis of Small Process Variations

$$T = \frac{PC}{1+PC}, \qquad \frac{dT}{T} = \frac{1}{1+PC}\,\frac{dP}{P} = S\,\frac{dP}{P}$$

$$S = \frac{1}{1+PC}, \qquad \frac{dS}{S} = \frac{-PC}{1+PC}\,\frac{dP}{P} = -T\,\frac{dP}{P}$$

$$G_{yd} = \frac{P}{1+PC}, \qquad \frac{dG_{yd}}{G_{yd}} = S\,\frac{dP}{P}$$

$$G_{un} = \frac{C}{1+PC}, \qquad \frac{dG_{un}}{G_{un}} = -T\,\frac{dP}{P}$$

Recall properties of S and T

- $S + T = 1$
- S small at low frequencies $S \approx 1$ at high frequencies
- T small at high frequencies $T \approx 1$ at low frequencies

$$M_s = \max_\omega |S(i\omega)|, \quad M_t = \max_\omega |T(i\omega)|$$

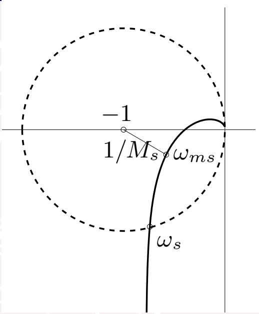
M_s - better than gain and phase margins

Gain margin

$$g_m \geq \frac{M_s}{M_s - 1}$$

Phase margin

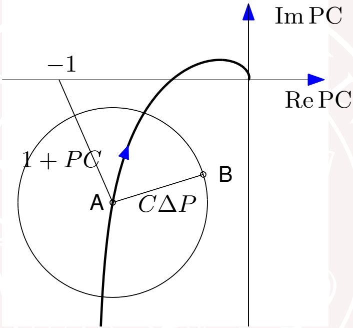
$$\varphi_m \geq 2 \arcsin \frac{1}{2M_s}$$



Constraints on both gain and phase margins can be replaced by constraints on M_s.

- $M_s = 2$ guarantees $g_m \geq 2$ and $\varphi_m \geq 30^\circ$
- $M_s = 1.6$ guarantees $g_m \geq 2.7$ and $\varphi_m \geq 36^\circ$
- $M_s = 1.4$ guarantees $g_m \geq 3.5$ and $\varphi_m \geq 42^\circ$
- $M_s = 1$ guarantees $g_m = \infty$ and $\varphi_m \geq 60^\circ$

Robustness margin, SISO case

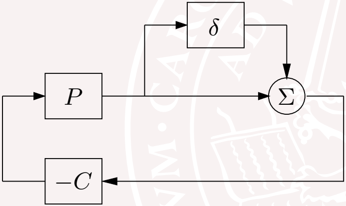


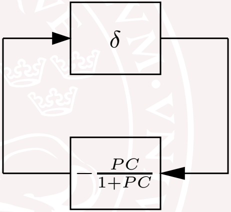
Stability for $P + \Delta P$ **guaranteed if** $\left|\frac{\Delta P}{P}\right| < \left|\frac{1 + PC}{PC}\right| = \frac{1}{|T|}$

Robustness problems when T is large, $M_t = \max |T(i\omega)|$

Robustness margin - multiplicative uncertainty

A feedback system where the process has multiplicative uncertainty, i.e. $P(1+\delta)$, where δ is the relative error, can be represented with the following block diagrams





The small gain theorem gives the stability condition

$$|\delta| < \left|\frac{1+PC}{PC}\right| = \frac{1}{|T|}$$

Same result as obtained before!

How about robustness for MIMO systems?

Spoiler alert: We'll start with a failed approach (MIMO Nyquist)

There is a generalization of the Nyquist theorem to the MIMO case

$$G(s) = W(s)\Lambda(s)W^{-1}(s)$$

Characteristic loci: $\lambda_i(s) :=$ eigenvalues of $G(s)$

Theorem [MIMO Nyquist]: If $G(s)$ has P_o unstable poles, then the closed loop system with return ratio $-kG(s)$ is stable if the characteristic loci of $kG(s)$ encircle the point -1 P_o times anticlockwise.

Why not use characteristic loci ?

$G(s)$ can have well behaved char. loci with great apparent stability margins to -1, but the loop can still be quite non-robust

What was finally agreed upon by control researchers:

Performance and robustness is best understood by using singular values $\sigma_i(G)$ instead of eigenvalues $\lambda_i(G)$

MIMO example - why use singular values

Plant model [Distillation column - Skogestad]

$$P(s) = \frac{1}{50s+1}\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}$$

Choose $C(s) = \frac{1}{s}P(s)^{-1}$ (dynamic decoupling), gives

Loop gain $PC = \frac{1}{s}I$

Closed-loop : $T(s) = PC(I+PC)^{-1} = \frac{1}{s+1}I$

Nice decoupled first order responses with time constant 1.

MIMO Nyquist diagram looks fine.

But non-robust...

Example -continued

In reality: 20 percent input uncertainty (e.g. valve variations)

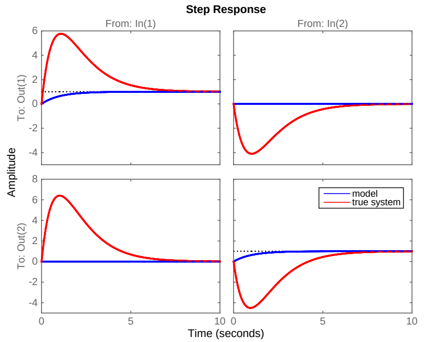
True control signal is $u_{i,p} = (1 + \delta_i)u_i$ with $|\delta_i| < 0.2$

$$P_\delta = \frac{1}{50s + 1} \begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix} \begin{pmatrix} 1.2 & 0 \\ 0 & 0.8 \end{pmatrix}$$

Using same controller as before gives

$$P_\delta C = \frac{1}{s} \begin{pmatrix} 14.83 & -11.06 \\ 17.29 & -12.83 \end{pmatrix}$$

Example -closed loop step responses



$$PC = \frac{1}{s}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{versus} \quad P_\delta C = \frac{1}{s}\begin{pmatrix} 14.83 & -11.06 \\ 17.29 & -12.83 \end{pmatrix}$$

With P: No interactions, nice step responses

With P_δ: Large interaction, 500 percent overshoot in step responses

Example -continued

The design is extremly sensitive to uncertainty on the inputs

But not to uncertainty on the outputs (easy to check)

Several indications of a directionality problem:

$$RGA(P) = \begin{pmatrix} 35 & -34 \\ -34 & 35 \end{pmatrix}$$

$$\mathrm{cond}(P) := \frac{\overline{\sigma}(P)}{\underline{\sigma}(P)} = 142$$

How do we analyse performance and robustness for MIMO systems ?

Failure of LQG to guarantee robustness

In the 70s there were great hopes to soon find THE robust control design method

For LQ design one automagically always got great robustness guarantees

$$M_s \leq 1$$

Disturbance rejection performance improved for all frequencies

Gain Margin $[1/2, \infty]$, Phase Margin ≥ 60 degrees.

Circle criterion: Stability under feedback with any nonlinear time-varying input gain with slopes in $(1/2, \infty)$.

(Requirements: No cross-terms, $Q_{12} = 0$. All states measurable.)

Robustness of LQG

Kalman filter producing $\hat{x}$ has similar (dual) robustness properties

Since the LQG controller combines two robust parts: LQ control and Kalman filtering, it was for a long time hoped that general robustness guarantees for the LQG controller would soon be found

But, output feedback $u = -L\hat{x}$ was surprisingly (?) found to have no automatic guarantees for robustness

This was a dissappointment, especially for people hoping to automize design

Turned attention towards robust control, e.g. H_∞ in the 80s

A new kid on the block

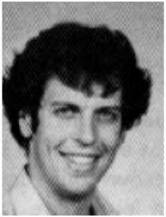
Honeywell Interoffice Correspondence

Date: August 23, 1977

To: C. A. Harvey

From: J. C. Doyle

Location: S&RC, Research



cc: L. Q. Gaussian
J. A. Hauge
A. P. Kizilos
A. F. Konar
E. E. Yore
N. R. Zagalsky
Systems and Control Technology

Subject: "Guaranteed Margins for LQG Regulators"

A B S T R A C T

There aren't any.

All engineers who have been using LQG methodology may pick up their Nichols charts from the supply room.

Robust stability vs robust performance

(Output) sensitivity function

$$S := (I + PC)^{-1}$$

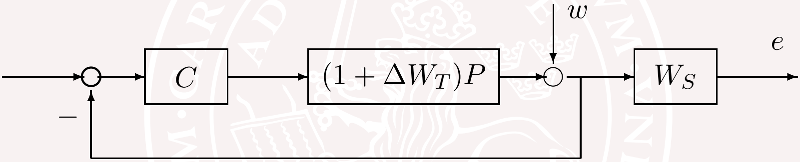
- Nominal stability(NS): S stable
- Nominal performance(NP): $\overline{\sigma}(S) \leq 1/|W_p|$, where $W_p(s)$ weight
- Robust stability(RS): $S_\delta := (I + P_\delta C)^{-1}$ stable, $\forall P_\delta \in \mathcal{P}$
- Robust performance(RP): $\overline{\sigma}(S_\delta) \leq 1/|W_p|$, $\forall P_\delta \in \mathcal{P}$

For SISO systems NP + RS $\Rightarrow$ RP (more or less, will show later)

For MIMO systems NP + RS $\not\Rightarrow$ RP

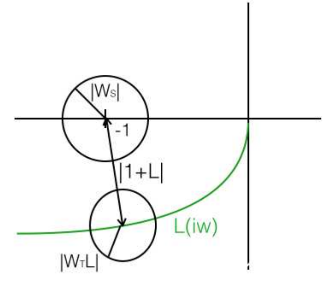
Multiviariable effects make simple analysis dangerous

Robust Performance - SISO case



Want $\|W_s S\| < 1$ for system with multiplicative uncertainty

Robust Performance - SISO case



Nominal Performance $\Leftrightarrow \|W_S S\|_\infty \leq 1$

Robust Stability $\Leftrightarrow \|W_T T\|_\infty \leq 1$

From figure:

$$\begin{aligned}\text{Robust Performance } &\Leftrightarrow |W_S| + |W_T L| \leq |1+L|, \quad \forall s = i\omega \\ &\Leftrightarrow |W_S S| + |W_T T| \leq 1, \quad \forall s = i\omega\end{aligned}$$

Robust Performance - SISO case

Robust Performance

$$\|T_{ew}\|_\infty < 1 \text{ for all } \|\Delta\| \le 1$$

is hence equivalent to the condition

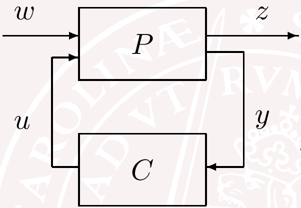
$$\max_\omega \left[\underbrace{|W_S S|}_{\text{nominal performance}} + \underbrace{|W_T T|}_{\text{robust stability}} \right] < 1$$

RP almost guaranteed when we have NP + RS

$$\boxed{\text{NP + RS} \Rightarrow \text{RP/2}}$$

Explains why RP is not a big issue for SISO systems

Highlight 1: H_∞ control



$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

$$T_{zw} = P_{11} + P_{12}C(I - P_{22}C)^{-1}P_{21}$$

Optimal control:

$$\min_{C-\mathrm{stab}} \|T_{zw}\|_\infty$$

Suboptimal control: Given γ find stabilizing C such that

$$\|T_{zw}\|_\infty < \gamma \qquad \Longleftrightarrow \qquad \|z\|_2 < \gamma\|w\|_2, \quad \forall w$$

The optimal control problem is solved by iterating on γ

The H_∞ norm = Induced L_2 norm

The H_∞ norm of a stable function $G(s)$ is given by

$$\|G\|_\infty = \sup_{\|u\|_2 \le 1} \|Gu\|_2 = \sup_\omega \|G(j\omega)\| = \sup_\omega \overline{\sigma}(G(j\omega))$$

For unstable $G(s)$ the norm is defined as $+\infty$

Notation:

$G \in RH_\infty^{p\times m}$ means $G(s)$ rational of size $p \times m$ with finite H_∞ norm

The H_∞ norm = Induced L_2 norm

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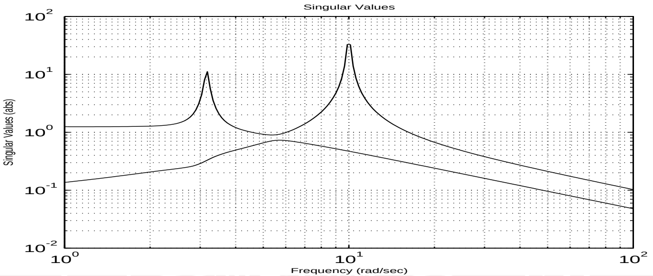
Singular value plot for 2×2 system



$$G(s) = \begin{bmatrix} \frac{10(s+1)}{s^2+0.2s+100} & \frac{1}{s+1} \\ \frac{s+2}{s^2+0.1s+10} & \frac{5(s+1)}{(s+2)(s+3)} \end{bmatrix}$$

The Matlab command `norm(G,'inf')` uses bisection together with the theorem above to get $\|G\|_\infty = 50.25$. Frequency sweep with 400 frequency points gives only the maximal value 43.53.

MIMO Performance

In the MIMO case the order of matrices matters

$$
\begin{aligned}
L_i &= CP, & L_o &= PC, \\
S_i &= (I + L_i)^{-1}, & S_o &= (I + L_o)^{-1}, \\
T_i &= I - S_i, & T_o &= I - S_o.
\end{aligned}
$$

TAT: Which of the following matrices are the same?

$$
PC(I+PC)^{-1}, \quad C(I+PC)^{-1}P, \quad (I+PC)^{-1}PC
$$

$$
(I+CP)^{-1}CP, \quad P(I+PC)^{-1}C, \quad CP(I+CP)^{-1}
$$

Highlight 2: When are Two Systems Close ?

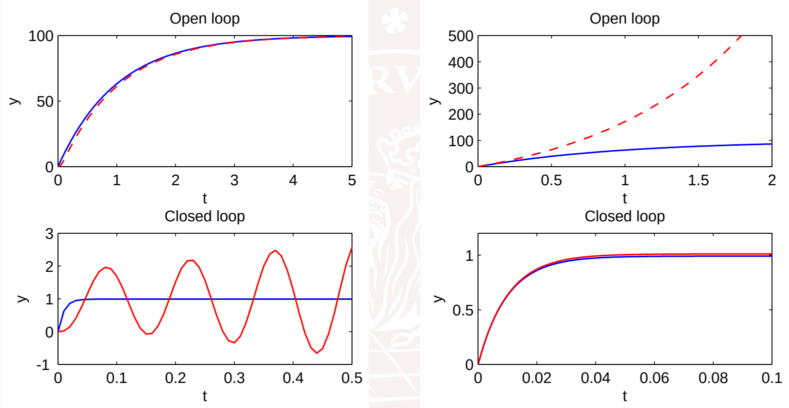
- For stable systems

$$\delta(P_1, P_2) = \max_{\omega} |P_1(i\omega) - P_2(i\omega)|$$

  as a measure of of closeness of two processes.
- Is this a good measure?
- Are there other alternatives?
- A long story

  Gap metric (Zames)
  Graph metric coprime factorization (Vidyasagar) $G = N/D$
  Vinnicombe's metric
  GOF performance metric

When are Two Systems Close?

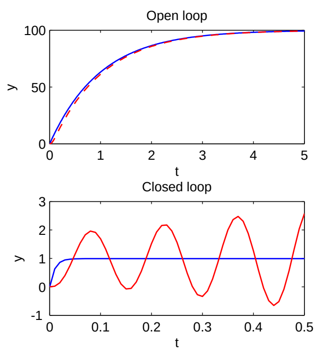


Comparing step responses can be misleading!

Frequency responses are better

Better to compare closed loop responses

Similar Open Loop Different Closed Loop



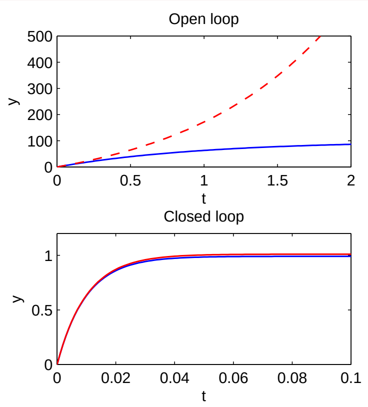
$$P_1(s) = \frac{100}{s+1}, \qquad P_2(s) = \frac{100}{(s+1)(1+0.025s)^2}$$

Complementary sensitivity functions with unit feedback $C = 1$

$$T_1 = \frac{100}{s+101}, \qquad T_2 = \frac{1.616e5}{(s+83.9)(s^2 - 2.90s + 1926s + 1926)}$$

Very different closed loop systems

Different Open Loop Similar Closed Loop



Systems and complementary sensitivity functions

$$P_1(s) = \frac{100}{s+1}, \quad T_1(s) = \frac{100}{s+101}, \quad P_2(s) = \frac{100}{s-1}, \quad T_2(s) = \frac{100}{s+99}$$

Closed loop systems are very similar

ν-Gap Metric [Vinnicombe]

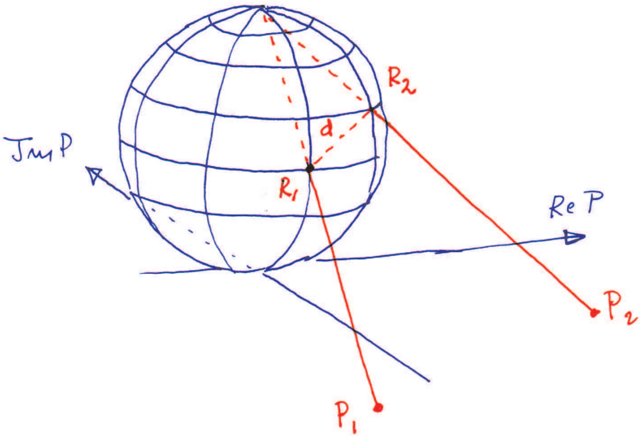
For the SISO case

$$\delta_\nu(P_1, P_2) = \sup_{\omega \in R} \frac{|P_1(j\omega) - P_2(j\omega)|}{\sqrt{1 + |P_1(j\omega)|^2}\sqrt{1 + |P_2(j\omega)|^2}}$$

(+ some ”winding number constraint”).

Geometrical interpretation: Distance on the Riemann sphere

Geometric Interpretation



Feedback Interpretation

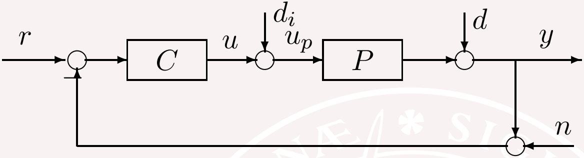
Consider systems with the transfer functions P_1 and P_2. Compare the complementary sensitivity functions for the closed loop systems obtained with a controller C that stabilizes both systems.

$$\delta(P_1, P_2) = \left| \frac{P_1 C}{1 + P_1 C} - \frac{P_2 C}{1 + P_2 C} \right| = \left| \frac{(P_1 - P_2) C}{(1 + P_1 C)(1 + P_2 C)} \right|$$

We have

$$\delta(P_1, P_2) \leq M_{s1} M_{s2} |(P_1 - P_2) C|$$

Vinnicombe's metric corresponds to $C = 1$, i.e. unit feedback.



$$
\begin{aligned}
y &= T_o(r-n) + S_oPd_i + S_od, \\
r-y &= S_o(r-d) + T_on - S_oPd_i, \\
u &= CS_o(r-n) - CS_od - T_id_i, \\
u_p &= CS_o(r-n) - CS_od + S_id_i
\end{aligned}
$$

1) Good disturbance rejection ”in all directions” if

$$\underline{\sigma}(L_o) >> 1, \quad \underline{\sigma}(C) \text{ sufficiently large}$$

2) Good robustness and good sensor noise rejection if

$$\overline{\sigma}(L_o) << 1, \quad \overline{\sigma}(L_i) << 1, \quad \overline{\sigma}(C) \text{ sufficiently small.}$$

Require 1) at low-frequency and 2) at high frequency

MIMO Loop-Shaping Design

A good performance controller design typically requires

- large gain in the low frequency region:

$$\underline{\sigma}(PC) >> 1, \quad \underline{\sigma}(CP) >> 1, \quad \underline{\sigma}(C) >> 1.$$

- small gain in the high frequency region:

$$\overline{\sigma}(PC) << 1, \quad \overline{\sigma}(CP) << 1, \quad \overline{\sigma}(C) \leq M$$

  where M is not too large.

Wouldn't it be nice to be able to do loopshaping worrying only about the gains and not care about phase and stability ?

MIMO requirements

Example: Weighted (output) sensitivity requirement

$$\|W_1(i\omega)S_o(i\omega)W_2(i\omega)\| \leq 1, \quad \forall\omega$$

Often $W_1(s), W_2(s)$ are chosen as rational functions without rhp poles or zeros

Matlab - General H_∞ design

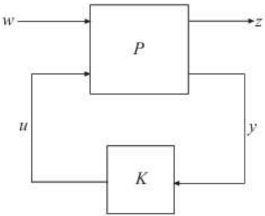
```
[K,CL,GAM,INFO] = hinfsyn(P,NMEAS,NCON)
```

`hinfsyn` computes a stabilizing H_∞ optimal `lti/ss` controller K for a partitioned `lti` plant P.

$$P = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

The controller, K, stabilizes the P and has the same number of states as P. The system P is partitioned where inputs to B_1 are the disturbances, inputs to B_2 are the control inputs, output of C_1 are the errors to be kept small, and outputs of C_2 are the output measurements provided to the controller. B_2 has column size (NCON) and C_2 has row size (NMEAS). The optional KEY and VALUE inputs determine tolerance, solution method and so forth.

The closed-loop system is returned in CL. This closed-loop system is given by `CL = lft(P,K)` as in the following diagram.



The achieved H_∞ cost γ is returned as GAM. The `struct` array INFO contains additional information about the design.

Matlab - mixsyn

```
[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3) or

mixsyn H-infinity mixed-sensitivity synthesis method for
robust control design. Controller K stabilizes plant G
and minimizes the H-infinity cost function
               ||   W1*S ||
               || W2*K*S ||
               ||   W3*T ||Hinf
where
    S := inv(I+G*K)         % sensitivity
    T := I-S = G*K/(I+G*K)  % complementary sensitivity
    W1, W2 and  W3 are stable LTI 'weights'
```

Example - mixsyn

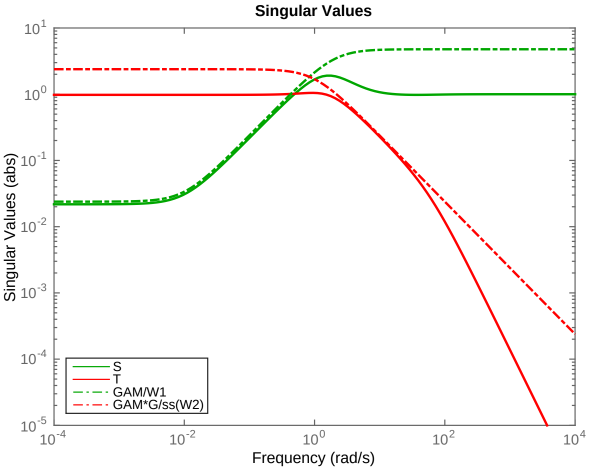
Minimizes H_∞ norm of

$$\begin{bmatrix} W_1(I+GK)^{-1} \\ W_2K(I+GK)^{-1} \\ W_3GK(I+GK)^{-1} \end{bmatrix}$$

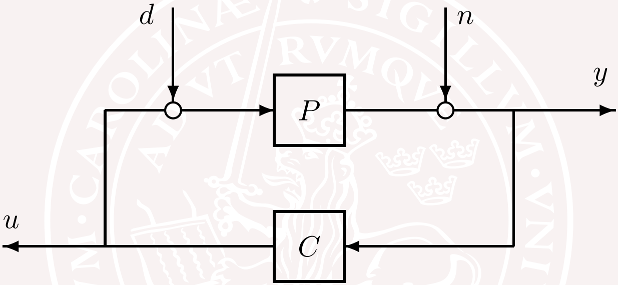
```
[C,CL,GAM,INFO] = mixsyn(G,W1,W2,W3)
```

```
G = (s-1)/(s+1)^2;
W1 = 5*(s+2)/(100*s+1);
W2 = 0.1;
[K,CL,GAM] = mixsyn(G,W1,W2,[]);
L = G*K;
S = inv(1+L);
T = 1-S;
sigma(S,'g',T,'r',GAM/W1,'g-.',GAM*G/ss(W2),'r-.')
```

Result

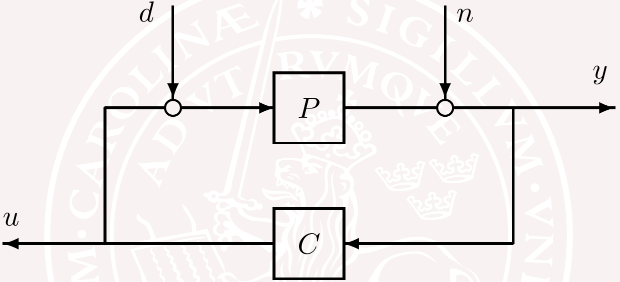

Singular Values
Singular Values (abs)
Frequency (rad/s)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
S
T
GAM/W1
GAM*G/ss(W2)

The GOF matrix



$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} I \\ C \end{bmatrix} (I + PC)^{-1} \begin{bmatrix} I & P \end{bmatrix} \begin{bmatrix} n \\ d \end{bmatrix}$$

Norm of GOF matrix



What is captured by the norm of the GOF matrix?

$$\left\| \begin{bmatrix} I \\ C \end{bmatrix} (I+PC)^{-1} \begin{bmatrix} I & P \end{bmatrix} \right\|_\infty \quad ?$$

GOF Stability Margin, $b_{P,C}$

Stability margin against ”normalised coprime perturbations” (we will learn what this is in the course)

$$b_{P,C} = \begin{cases} \left\| \begin{bmatrix} I \\ C \end{bmatrix} (I + PC)^{-1} \begin{bmatrix} I & P \end{bmatrix} \right\|_\infty^{-1} & \text{if } C \text{ stabilizes } P \\ 0 & \text{otherwise} \end{cases}$$

The larger $b_{P,C} \in [0, 1]$ is, the more robustly stable the closed loop system is.

Remark: Note that $b_{P,C}$ depends on scalings of inputs and outputs.

Good fit with Vinnicombe's metric:

Theorem Assume (P_1, C) is stable, then

$$\delta_\nu(P_1, P_2) < b_{P_1,C} \quad \Longrightarrow \quad (P_2, C) \text{ is also stable.}$$

Matlab - loopsyn

Syntax

```
[K,CL,GAM,INFO]=loopsyn(G,Gd)
[K,CL,GAM,INFO]=loopsyn(G,Gd,RANGE)
```

Description

`loopsyn` is an H_∞ optimal method for loopshaping control synthesis. It computes a stabilizing H_∞controller K for plant G to shape the `sigma` plot of the loop transfer function GK to have desired loop shape G_d with accuracy γ = GAM in the sense that if ω_0 is the 0 db crossover frequency of the `sigma` plot of $G_d(j\omega)$, then, roughly,

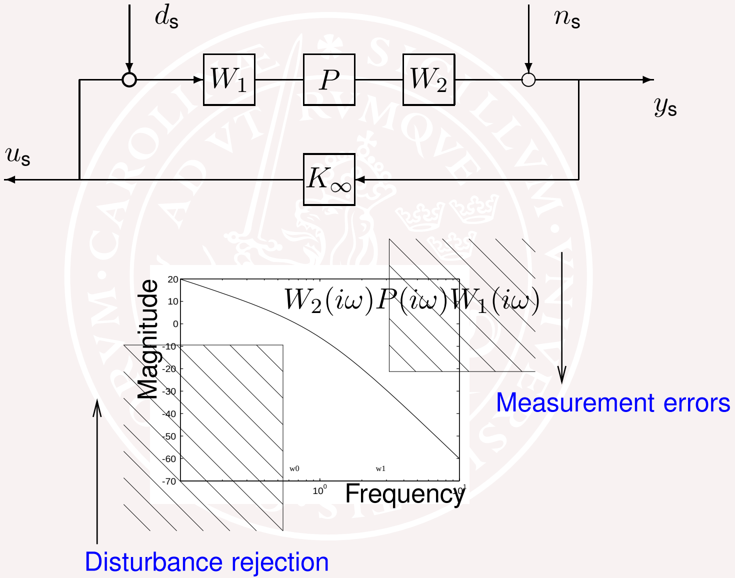
$$\underline{\sigma}(G(j\omega)K(j\omega)) \geq \frac{1}{\gamma}\, \underline{\sigma}(G_d(j\omega)) \text{ for all } \omega < \omega_0 \tag{1-14}$$

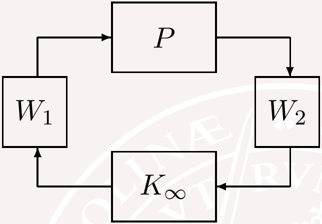
$$\overline{\sigma}(G(j\omega)K(j\omega)) \leq \gamma\, \overline{\sigma}(G_d(j\omega)) \text{ for all } \omega > \omega_0 \tag{1-15}$$

The STRUCT array `INFO` returns additional design information, including a MIMO stable min-phase shaping pre-filter W, the shaped plant $G_s = GW$, the controller for the shaped plant $K_s = WK$, as well as the frequency range $\{\omega_{min},\omega_{max}\}$ over which the loop shaping is achieved

Input Argument	Description
G	LTI plant
Gd	Desired loop-shape (LTI model)
RANGE	(optional, default `{0,Inf}`) Desired frequency range for loop-shaping, a 1-by-2 cell array $\{\omega_{min},\omega_{max}\}$; ω_{max} should be at least ten times ω_{min}

Hihglight 3: Glover McFarlane Loopshaping





1) Choose W_1 and W_2 and absorb them into the nominal plant P to get the shaped plant $P_s = W_2 P W_1$.
2) Calculate $b_{opt}(P_s)$. If it is small (< 0.25) then return to Step 1 and adjust weights.
3) Select $\epsilon \leq b_{opt}(P_s)$ and design the controller K_∞ such that

$$\left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + P_s K_\infty)^{-1} \tilde{M}_s^{-1} \right\|_\infty < \frac{1}{\epsilon}.$$

4) The final controller is $K = W_1 K_\infty W_2$.

Other Highlights

- The game theory interpretation of H_∞ control
- and the connection to risk-aversive control
- QFT design
- Some recent research results
- ...

What now?

- Self-study material - see home page
- Exercise 1
- Handin 1