# Handin 4

**1.** At the end of the lecture we saw that although

$$P_1(s) = \frac{200}{2s+1}, P_2(s) = \frac{200}{2s-1},$$

had very different open loop step responses, in closed loop they behaved in a similar fashion. Justify this claim by computing $\delta_\nu(P_1, P_2)$ (use the Matlab function `gapmetric` or write your own). One might then be tempted to think that if the $\nu$-gap between two systems is large, then in closed loop the behaviour of the two systems would be different. Refute this claim by designing a single controller that achieves both $b_{P_1,C} > 0.3$ and $b_{P_3,C} > 0.3$, where

$$P_3(s) = \frac{100}{(s+1)^2}.$$

Explain why this refutes the claim, and also in what sense a large value of $\delta_\nu(P_1, P_3)$ does imply that the closed loop behaviour of the two systems $P_1, P_3$ is different.

**2.** Consider again the distillation column process from Lecture 1

$$P(s) = \frac{1}{50s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}.$$

Design a controller with integral action that achieves a control bandwidth of around 0.01 rad/s and high levels of robustness to unmodelled dynamics for high frequencies. Now consider the perturbed plant

$$P_\delta(s) = P(s)\begin{bmatrix} 1.2 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

In Lecture 1 we saw that a naively designed controller performed appallingly on this $P_\delta(s)$ (this was used to illustrate that SISO intuition can be very misleading for MIMO problems). Compute $\delta_\nu(P(s), P_\delta(s))$. Does this go some way to explaining why this is the case? Check the step responses of your controller when applied to the perturbed plant. Would you say they are satisfactory? If not, iterate your design.