# Exercise session 3

*Simple Uncertainty Models. Unstructured Uncertainties. Small Gain Theorem and Robust Stability. Robust Performance. Linear Fractional Transformation.*

## Reading Assignment

[Zhou, ch.8,9]
Optional: [Doyle,Francis,Tannenbaum, ch.4], [Skogestad,Postlethwaite ch.7,8]

## Exercises

**E3.1** [Zhou] 8.1

**E3.2** [Zhou] 8.3
Using Small Gain Theorem find a condition for $K$ to be robustly stabilizing.

**E3.3** [Zhou] 8.7

**E3.4** [Zhou] 8.9

**E3.5** [Zhou] 8.13

**E3.6** Assume that the nominal plant is a double integrator $P_0(s) = 1/s^2$. The performance requirement is that the plant output should track reference inputs over the frequency range $[0, 1]$. The performance weight could therefore be chosen as a Butterworth filter. Choose a third order Butterworth filter with cutoff frequency 1 rad/s. Take the uncertainty model to be

$$P_\Delta = (1 + \frac{0.21s}{0.1s + 1}\Delta)P_0, \quad \|\Delta\|_\infty \leq 1.$$

(a) Design a proper controller the achieve nominal stability.

(b) Check if this controller gives robust stability also. If not, repeat until it does.

(c) Compute a robust performance level which is the maximal value of the performance over the set $\|\Delta\|_\infty \leq 1$. Compare this level with the nominal performance.

## Hand-In problems

**H3.1** [Zhou] 8.12

**H3.2** [Zhou] 9.3