Exercise 1

Introduction. The class of linear systems as a linear space. Norm and inner product as a way to measure distance. Banach and Hilbert spaces. The Lebesgue spaces L_2 and L_∞. The Hardy spaces H_2 and H_∞.

Reading Assignment

- Refresh in memory and elaborate the unknown details about necessary linear algebra and linear system facts in [Zhou] Ch. 2 and 3.
- Read [Zhou] Ch. 4. Optional reading: [Doyle,Francis,Tannenbaum] Ch. 2.

1. [Zhou] 2.5–2
2. Suppose that u is a continuous function whose derivative is continuous too. Which of the following qualifies as a norm for u?
   (a) $\sup_t |\dot{u}(t)|$,
   (b) $|u(0)| + \sup_t |\dot{u}(t)|$,
   (c) $\max\{\sup_t |u(t)|, \sup_t |\dot{u}(t)|\}$,
   (d) $\sup_t |u(t)| + \sup_t |\dot{u}(t)|$.
3. Prove that the relation
   $$\langle f, g\rangle_2 = \int_{-\infty}^{\infty} \operatorname{tr}\left[g(t)^* f(t)\right] dt$$
   satisfies all axioms of the inner product.
4. [Zhou] 4.3
5. For the linear system $y = Gu$ prove that
   $$\|G\| = \sup_{\|u\|=1} \|y\|.$$
   (Note the equality sign instead of inequality.)
6. Calculate the H_∞ distance between two plants
   $$G_1(s) = \frac{1}{s+1}, \qquad G_2(s) = \frac{1}{s+1}e^{-\theta s}$$
   for $\theta = \{0.01, 0.1, 1\}$. How does the distance depend on θ?

Hand-In problems:

1. [Zhou] 4.4
2. [Zhou] 4.8