Control Paradigms

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Control Paradigms - Bottom Up Design

1. Introduction
2. Linear Structures
3. Cascade Control
4. Mid-range and Split-range
5. Nonlinear Structures
6. Selectors
7. Architecture
8. Summary

Theme: Brick by brick.

Architecture

- Architecture greek arkhitekto (arkhi-chief) tekton (builder) includes design, planning and construction
- Design is a part of all engineering disciplines: A, ME, EE, CE, CS, AA, ...
- Building complex systems from standard parts have been a standard procedure. Nuts, bolts standard assemblys. Transistors, boards, cabinetts. VLSI, graphs, design rules, libraries. Subroutines, programs, component software.
- Many attempts to make *design theories* and *design methods*
- CS has been an interesting proving ground because it easy to experiment, but also easy to include realistic settings
- Chip design is the role model

  Abstractions, Layering, Design rules, Testing
- Can we imitate it?

Issues and Work Flow

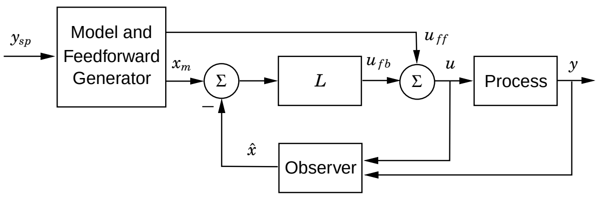
Key considerations
- Performance
- Cost
- Stability
- Robustness
- Optimality
- Reliability
- Ease of use

Engineering steps
- Requirements
- Specifications
- Modeling
- Analysis
- Simulation
- Design
- Implementation
- Commissioning
- Operation
- Upgrading

Top-Down Design or Bottom-Up

Bottom up: Build the system from components
- What components?
- Principles for combining them

Top down: Look at the total problem and solve it



Building Blocks and System Principels

Buiding blocks
- Linear

  Controllers
  Filters
- Nonlinear

  Limiters
  Switches
  Selectors
  Logic
- Estimators
- Optimizers

System principles
- Feedback
- Feedforward
- Cascade
- Midranging
- Model following
- Selectors
- Gain scheduling
- Adaptation

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Linear Structures

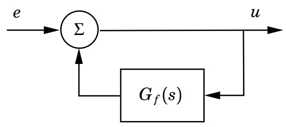
- Controllers and filters
- Generalized integral control
- Feedforward
- Posicast control
- Model following
- Cascade control
- Mid- and split range
- Repetitive control
- Complementary filtering
- Observers and state feedback

Generalized Integral Control

- Integral control was a real break through
  Maxwell (1868) mentioned that Siemens (1866) had distinguished between governors (PI) and moderators (P)
- Automatic removal of steady state errors (automatic reset)
- Integral control eliminates a disturbance that is constant with unknown amplitude
- Can it be generalized to other types of disturbances?
  Ramps $v(t) = a_0 + a_1 t$ where a_0, a_1 are unknown
  Jerks $v(t) = a_0 + a_1 t + a_2 t^2$ where a_0, a_1 a_2 are unknown
  Sinusoidal disturbances with known frequency but unknown amplitude and phase
  Sinusoidal disturbances with unknown frequency, amplitude and phase
  Periodic distrubances with known period
- Idea: Build a model of the disturbance in the controller!

Generalized Integral Control

- Constant but unknown
- Ramps with unknown levels and rates
- Sinusoidal with known frequency but unknown amplitude and phase
- Periodic with known period but unknown shape



$$C(s) = \frac{k}{1 - G_f(s)}$$

$$G_f(s) = \frac{1}{1+sT_f} \qquad C_{const}(s) = 1 + \frac{1}{sT}$$

$$G_f(s) = \frac{2\zeta\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \qquad C_{sinusoid}(s) = \frac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{s^2 + \omega_0^2}$$

$$G_f(s) = e^{-sL} \qquad C_{periodic}(s) = \frac{1}{1 - e^{-sL}}$$

Elimination of Periodic Disturbances

$$G_f(s) = e^{-sL} \qquad C_{periodic}(s) = \frac{1}{1 - e^{-sL}}$$

Control law

$$u(t) = e(t) + u(t - L)$$

Transfer function from disturbance to output

$$G_{yd}(s) = \frac{P(s)}{1 + P(s)C(s)} = \frac{P(s)(1 - e^{-sL})}{1 - e^{-sL} + P(s)}.$$

The relation between load disturbance and output

$$(1 - e^{-sL} + P(s))Y(s) = P(s)(1 - e^{-sL})D(s).$$

Notice that the time function corresponding to $(1 - e^{-sL})D(s)$ is $d(t) - d(t - L)$, which is zero if d has period T. Compare with PI control.

Recovering Robustness

- No difficulties with infinite gain for a PI controller
- Difficulties with controller that have infinite gain at high frequencies
- Remedy: reduce largest gain and introduce high frequency roll-off

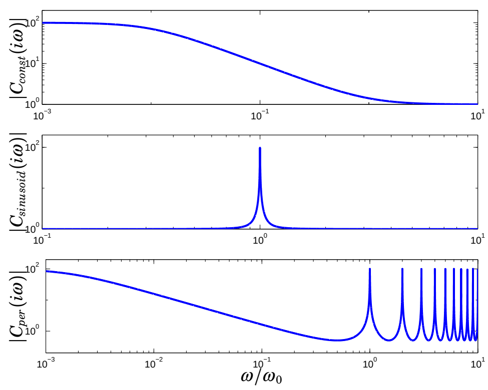
Replacing G_f by αG_f gives $C = \frac{1}{1-\alpha G_f}$

$$C_{\text{const}}(s) = \frac{1 + sT}{1 - \alpha + sT}$$

$$C_{\text{sinusoid}}(s) = \frac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{s^2 + 2(1-\alpha)\zeta\omega_0 s + \omega_0^2}$$

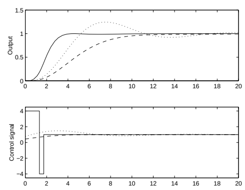
$$C_{\text{per}}(s) = \frac{1}{1 - \alpha e^{-sT}}$$

Bode Plots for $\alpha = 0.99$



Design of Feedforward ...

- Essentially a problem of computing inverses or approximate inverses of systems $PM_u = M_y$.
- Make reasonable demands, time delays and RHP zeros of P must be included in M_y.
- The feedforward parts M_u and M_y can be nonlinear. Modelica can deliver inverse models.



PI control with $M_s = 1.4$ (dashed) and $M_s = 2.0$ (dotted) for $P(s) = 1/(s+1)^4$ and nonlinear feedforward (solid)

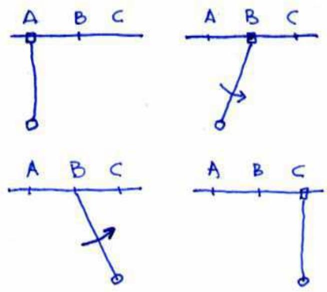
Flatness - Michel Fliess

- The concept of flat output
- A flat output y is an output signal such that the state and x the input u can be generated from y and its derivatives.

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = b_n u$$

- Insight: Some combinations of the state are easy to manipulate, clear what restriction must be imposed on desired response y_m
- Drawback: Are these signals close to the ones we are interested in?
- Useful for feed-forward design by making the flat output behave in the desired way
- Can be applied to nonlinear systems

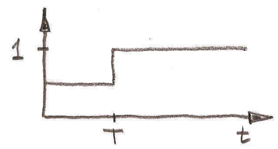
Moving a Hanging Container - Posicast Control



- O. J. M. Smith Posicast control of damped oscillatory systems, Proc. IRE. (45) 1957, 1249-1255
- Has been used succcessfully for cranes and micro systems
- What is the transfer function?

Feedforward Transfer Function

Step response



- Transfer function for posicast control
$$G_{ff}(s) = \frac{1}{2}(1 + e^{-sT}).$$
- Sinusoidal signals of frequencies $\omega = \omega_0, 3\omega_0, 5\omega_0 \ldots$, where $\omega_0 = 2\pi/T$ are cancelled
- Nonrational transfer function
- Easy to implement posicast control using digital control Richards bio talk at LCCC

Control of Oscillatory Systems

Process model

$$P(s) = \frac{\omega_0^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2}$$

Integrating controller with error feedback and posicast feedforward

$$C(s) = \frac{k_i}{s}(\gamma + (1-\gamma)e^{-sT_d})$$

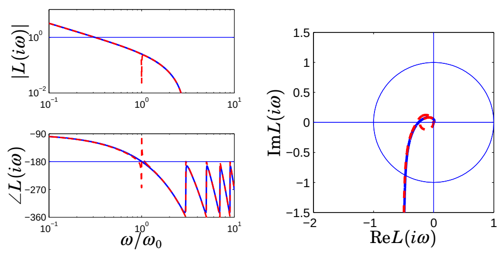
$$\gamma = \left(1 + e^{-\zeta_0\pi/\sqrt{1-\zeta_0^2}}\right)^{-1}, \qquad T_d = \frac{\sqrt{1-\zeta_0^2}}{\omega_0}.$$

Loop transfer function is (robustness valley: $\mathrm{Re}L(i\omega) = -0.5$)

$$L(s) = \frac{k_i(\gamma + (1-\gamma)e^{-sT_d})\omega_0^2}{s(s^2 + 2\zeta_0\omega_0 s + \omega_0^2)} \approx \frac{k_i}{s} - k_i\frac{(1-\gamma)\pi + 2\zeta_0}{\omega_0}$$

$$k_i = 0.5\frac{\omega_0}{(1-\gamma)\pi + 2\zeta_0}.$$

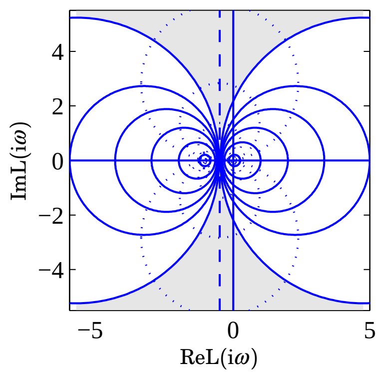
Nyquist and Bode Plots



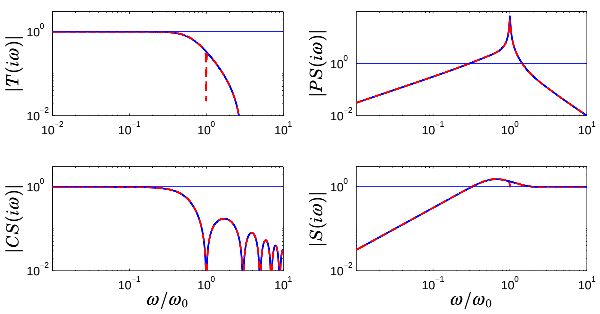
- Parameters: $\zeta_0 = 0.01$, $k_i = 0.32\omega_0$
- Performance: $M_s = 1.51$, $M_t = 1.00$, $M_{ps} = 66.5$, $M_cs = 1.00$ and $\omega_B = 0.62\omega_0$

The Robustness Valley

$$|\Delta P/P| < 1/|T|$$



Gain Curves for GoF



Compare with PI controller with notch filer
in Lecture on Poleplacement
There is also a 2DoF version

Complementary Filtering

- A technique to combine information from several sensors
- A precursor to Kalman filtering
- Useful in its own right
- Requires only models of sensor systems

Signal model (y_1 slow but accurate, y_2 fast but drifting)

$$y_1 = x + n_1, \qquad y_2 = x + n_2$$

Filter for recovering the variable x

$$X_f(s) = G_1Y_1 + G_2Y_2 = \frac{1}{s+1}Y_1(s) + \frac{s}{s+1}Y_2(s)$$

with $G_1 + G_2$, G_1 low pass filter, G_2 high pass.

A Kalman Filter Solution

Model the measured value x_1 and the drift of the second sensor as unknown constants

$$y_1 = x_1 + n_1, \qquad y_2 = x_1 + x_2 + n_2, \qquad \dot{x}_1 = 0, \qquad \dot{x}_2 = 0$$

The Kalman filter

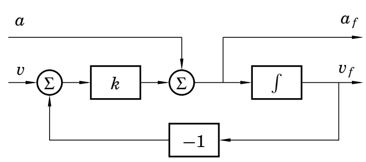
$$\frac{d}{dt}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{pmatrix}k_{11}(y_1 - \hat{x}_1) + k_{12}(y_2 - \hat{x}_1 - \hat{x}_2)\\ k_{21}(y_1 - \hat{x}_1) + k_{22}(y_2 - \hat{x}_1 - \hat{x}_2)\end{pmatrix}$$

After some calculations

$$\hat{X}_1(s) = \frac{k_{11}s + k_{11}k_{22} - k_{12}k_{21}}{s^2 + (k_{11} + k_{22} + k_{12}) + k_{11}k_{22} - k_{12}k_{21}}Y_1(s) + \frac{k_{12}s}{s^2 + (k_{11} + k_{22} + k_{12}) + k_{11}k_{22} - k_{12}k_{21}}Y_1(s)$$

Velocity and Acceleration Measurements

Determine estimates of velocity and acceleration from measurements of the same quantities



$$V_f(s) = \frac{s}{s+k}V(s) + \frac{k}{s+k}V(s)$$

$$V_f(s) = \frac{1}{s+k}A(s) + \frac{k}{s+k}V(s)$$

$$A_f(s) = \frac{s}{s+k}A(s) + \frac{ks}{s+k}V(s)$$

Use of an accelerometer and a rate gyro to determine tilt for the Segway is a similar problem.

Bengt Sjöberg Saab

I tidigare projekt hade man ju stött på behovet av filter, speciellt för att ta hand om brusiga radarsignaler. Man upptäckte då att tex antennvinklarna från egen flygradar mot ett radarföljt mål på grund av målets och det egna flygplanets fart, accleration och rotation vairerade starkt på grund av grundläggande kinematiska samband. ... Jag lärde mig ju snart att inse att dessa praktiska åtgärder helt enkelt bottnade i att man måste tvinga sina filtrerade variabler att satisfiera en modell för sambanden mellan accelerationer, farter och positioner hos eget flygplan och mål. Dessa modellsamband sattes då upp if vektorform varvid det oftast visade sig praktiskt att arbeta i olika koordinatsystem som oftast roterade. ... På detta sätt uppstod vad jag då efter viss vånda valde att kalla “komplementära filter”.

Complementary Filters Observers

Both

- Generate estimates of signals that are not measured directly
- Unify information from different sensors (sensor fusion)
- Can be optimized if noise information is available

Complementary filters

- Require models of sensor systems only not process dynamics

Observers

- Require models of process dynamics that typically involves command signals.
- Process inputs provide phase lead.

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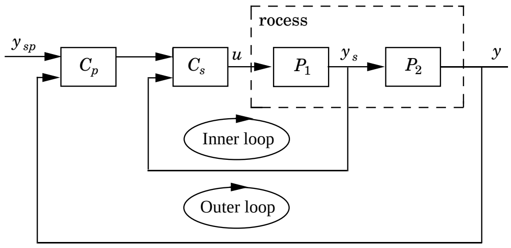
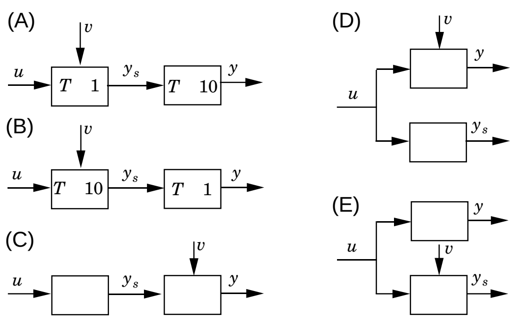
Cascade Control



- Incorporating several sensors
- State feedback ultimate case
- Tight feedback around disturbances and uncertainty
- Linearize a nonlinear actuator
- Integral action and windup

When is Cascade Control Useful?

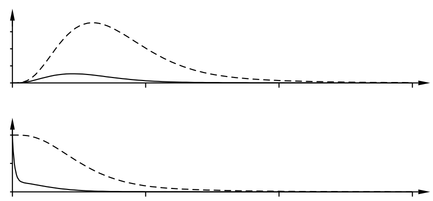


Cascade Control - Example

Process dynamics

$$P_1(s) = \frac{1}{s+1} \qquad P_2(s) = \frac{1}{(s+1)^3} \qquad P(s) = P_1P_2(s) = \frac{1}{(s+1)^4}$$

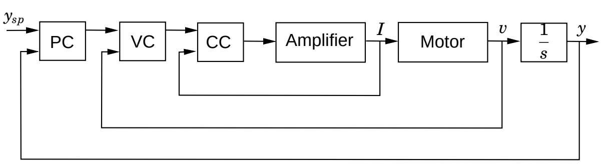
- PI Controller outer loop $K = 0.37$, $T_i = 2.2$
- P Controller inner loop $K = 5$, PI outer $K = 0.55$, $T_i = 1.9$



Introduce integral action in inner loop

Examples of Cascade Control

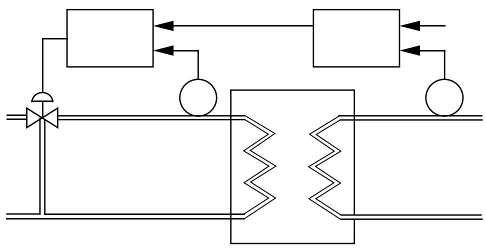
Motordrive



Three cascaded loops

- Current loop
- Velocity loop
- Position loop

Control of Heat Exchanger

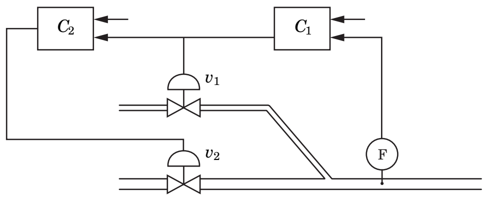


- Output temperature is the primary variable
- Three-way valve
- Flow measurement is used to mix primary water

Control Paradigms - Bottom Up Design

Theme: Brick by brick.

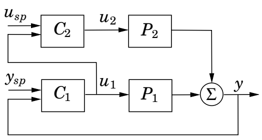
Mid-Range Control or Dual Actuation



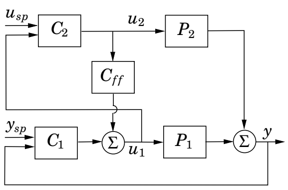
- Use parallel actuators to obtains high actuation precision and wide actuation range
- Fine actuation through v_1, course actuation through v_2
- Try to keep the valve v_1 in the mid range
- Course actuation can also be discrete (chillers)
- Separate time scales

Mid-Range Control or Dual Actuation ...

Basic structure

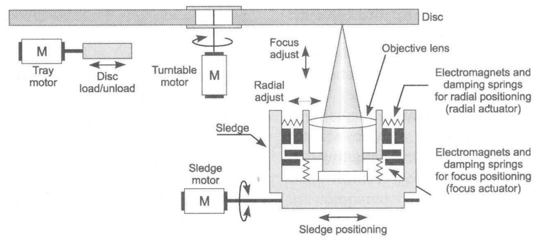


With feedforward



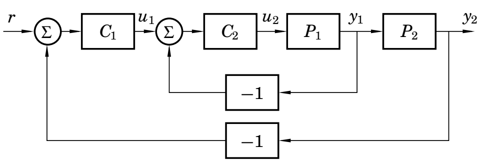
- The primary control variable u_1 controls y with limited actuation range
- The secondary controller adjusts the coarse control u_2 to keep u_1 in range

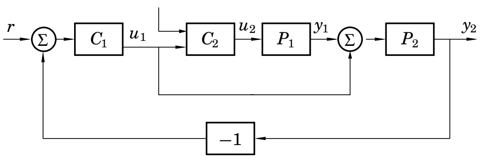
CD Player



- The optical package is light with a voice coil drive
- The sledge drive is slower and does the coarse motion

Cascade and Mid-range are Duals PH

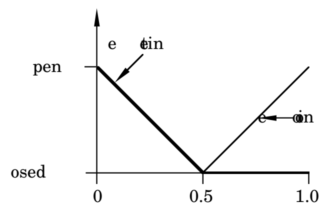




Switch sum and split and reverse signal flow!

Split Range Control

Usign one controller for two actuators, typically heating and cooling.



Actually nonlinear but it fits here anyway

Bottom-Up Design of Complex Systems

Theme: Brick by brick.

Nonlinear Structures

- Linearization: feedback, jitter
  - Pre- and post compensators
  - Feedback
  - Feedback linearization
  - Jitter
- Limiters
- Ratio control
- Controllers with logic
- Gain scheduling
- Adaptation

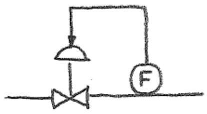
Linearization

- Both sensors and actuators can be linearized in open loop by feedforward
- Feedback can also be used effectively when sensors are available

Open loop

Process model $y = f(u)$
Feedfoward: $u = f^{-1}(u_c)$
Hence y=$f(f^{-1}(u_c)) = u_c$
Requires model
Sensitivity =1

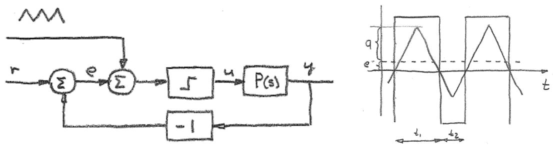
Feedback



Requires sensor
Less sensitive

Linearization by Using Jitter Signals

Mechanical and electrical jitter



When a triangular jitter signal is added to the error signal the average relay output is

$$\frac{T_p}{4}\left(1+2\frac{e}{a}\right) - \frac{T_p}{4}\left(1-2\frac{e}{a}\right) = \frac{e}{a}$$

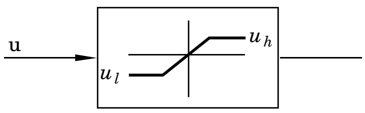
The combination of a relay with a jitter signal thus acts like a saturated linearity

Limiters

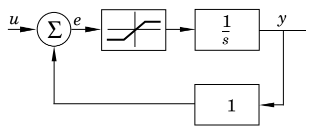
Limiters are used to avoid windup and to limit levels and rates for command signals (never ask the system to do more than it can). Kurt Nicolin Asea (legendary Swedish industrialist): “To add more workorders to an overloaded production unit increases confusion but not productivity.”

- Avoid actuator saturation
- Match demand to process capabilities
- Windup protection

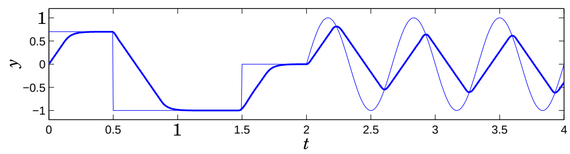
A simple limiter



Simple Rate Limiter

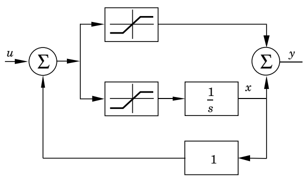


Behavior, notice that it creates phase lag

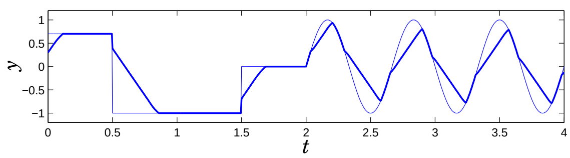


The JAS Gripen problem show video

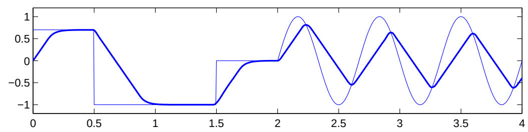
Jump and Rate Limiters

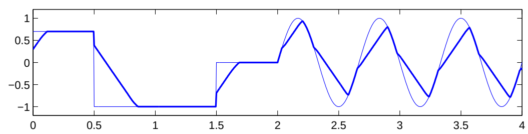


Behavior, less phase lag than with rate limiter



Rate and Jump-and Rate Limiters

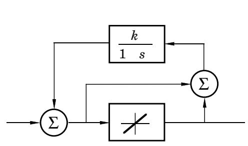


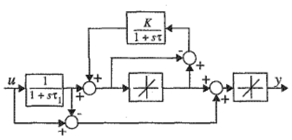


Rate limiters give phase lag, JAS Gripen
Jump-and-rate limiters are commonly used in power systems

Lars Rundqwist’s JAS Gripen Fix

- Assignment of authority for manual and automatic control
- Rate saturation in hydraulic servos
- Rate saturation causes phase lag
- Commissioning of flight control systems
- Inspiration from integral windup

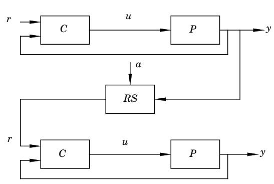




Ratio Control

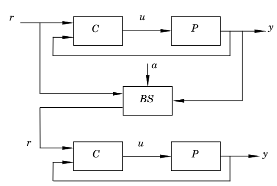
A common problem is to mix flows in given proportions. Ratio controllers is one way to do this selector control is an alternative (see Air-Fuel control later)

Ratio control



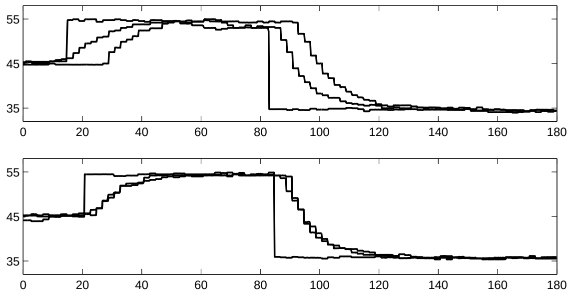
$r_2 = a y_1$

Tores blend station



$r_2(t) = a\left(\gamma r_1(t) + (1-\gamma) y_1(t)\right)$

Tores Blend Station (real data)

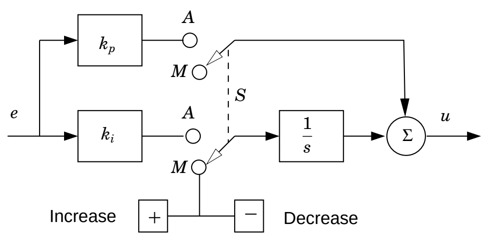


Top curves ratio control, bottom curves blend station

Surge Tank Control

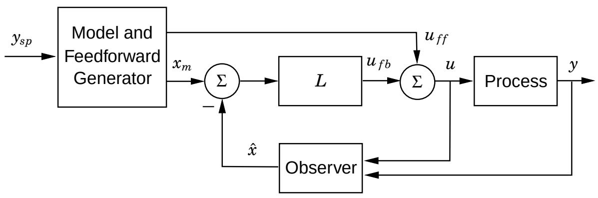
- Buffer tanks are used to smooth production rates
- Tanks should neither be empty nor overflow
- Not a conventional level control problem where it is attempted to keep the level constant
- If the levels are constant the buffer tanks can simply be removed
- Anna!!

Switching Hand-Automatic



Controllers with Tracking Mode

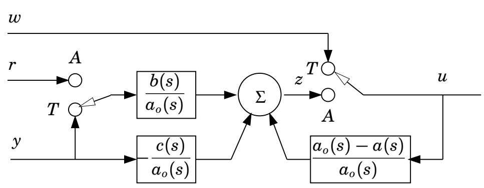
Trivial with the observer representation



Rewrite in tracking form

$$a(s)U(s) = b(s)R(s) - c(s)Y(s)$$
$$a_o(s)U(s) = b(s)R(s) - c(s)Y(s) + \big(a_o(s) - a(s)\big)$$

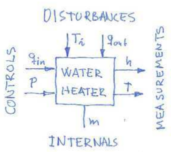
Controllers with Tracking Mode



Parameter Variations

- Robust control
  Find a control law that is insensitive to parameter variations
- Gain scheduling
  Measure variable that is well correlated with the parameter variations and change controller parameters
- Adaptive control
  Design a controller that can adapt to parameter variations
- Many different schemes
  Model reference adaptive control
  The self-tuning regulator
  L_1 adaptive control (later in LCCC)
- Dual control
  Control should be directing as well as investigating!

A Water Heater



Mass and energy balances

$$\frac{dm}{dt} = q_{in} - q_{out}, \; m = \rho A h$$
$$\frac{d(cmT)}{dt} = P + c q_{in} T_{in} - c q_{out} T$$

Traditional - Linearize

$$\frac{dm}{dt} = q_{in} - q_{out} \qquad m = \rho A h$$
$$\frac{dT}{dt} = \frac{q_{in}}{m}(T_{in} - T) + \frac{1}{cm} P$$
$$P = k_p cm(T_0 - T) + k_i cm \int^t (T_0 - T)dt - c q_{in}(T_{in} - T)$$
$$P = k_p c\rho A h(T_0 - T) + k_i c\rho A h \int^t (T_0 - T)dt - c q_{in}(T_{in} - T)$$

Energy Control a la Krister Forsman

A water heater

$$\frac{dm}{dt} = q_{in} - q_{out}$$
$$\frac{dQ}{dt} = \frac{d(cmT)}{dt} = P + c q_{in} T_{in} - c q_{out} T$$
$$m = \rho A h$$

Control law

$$P = k_p(Q_0 - Q) - c(q_{in} T_{in} - q_{out} T) + k_i \int^t (Q_0 - Q)dt$$
$$k_p = 2\zeta\omega_c, \qquad k_i = \omega_c^2$$
$$P = k_p c\rho A h(T_0 - T) + k_i \rho A \int^t h(T_0 - T)dt - c(q_{in} T_{in} - q_{out} T)$$
$$P = k_p c\rho A h(T_0 - T) + k_i c\rho A h \int^t (T_0 - T)dt - c q_{in}(T_{in} - T)$$

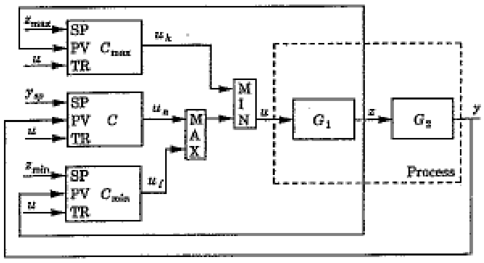
Gain scheduling from physics! Trivial design!

Control Paradigms - Bottom Up Design

1. Introduction
2. Bottom-Up or Top-Down
3. Linear Structures
4. Cascade Control
5. Mid-range and Split-range
6. Nonlinear Structures
7. Selectors
8. Architecture
9. Summary

Theme: Brick by brick.

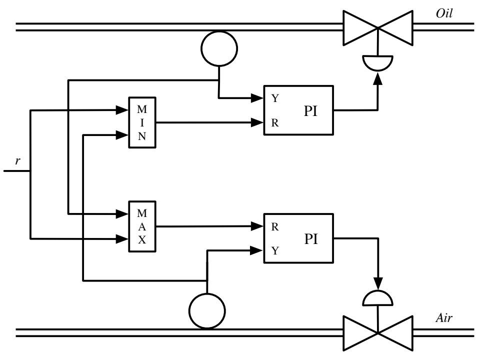
Selector Control



- Control with constraints
- Mixing objectives
- Logic and hybrid
- Median selectors
- Two-out-of-three
- Windup protection

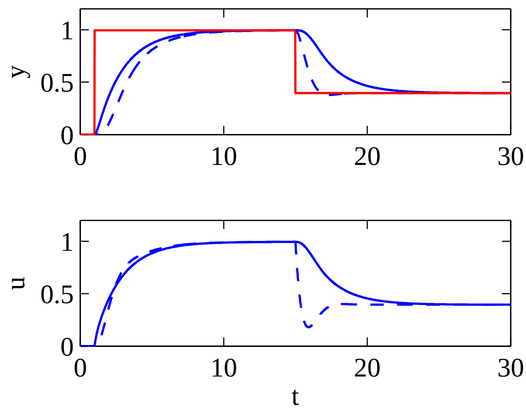
Stability Analysis- Piecewise linear systems?

Air-fuel Controller



- Make sure the boiler always runs with excess oxygen
- Discuss increase and decrease in power demand
- Compare with use of logic

Simulation



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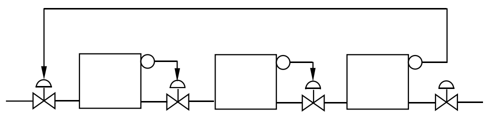
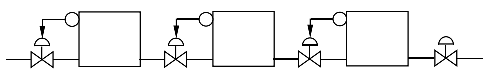
Architecture

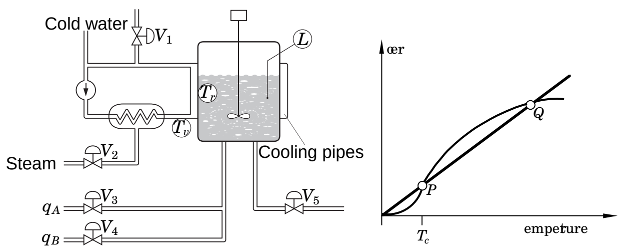
Control in the direction of the flow or opposite



Control in the direction opposite to the flow

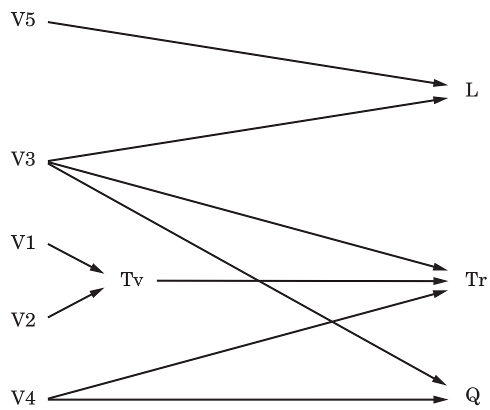


System Structuring



- Develop static model
- Understand the physics
- Find controller structure P&I diagram
- Simple process dynamics

Dependencies



Control Paradigms - Bottom Up Design

Theme: Brick by brick.

Summary

- A rich collection of methods and ideas
- Generalized integral control
- Cascade control and Midranging (duals)
- Delay related, Smith predictors, posicast
- Internal model control
- Selectors
- Can it be structured and formalized?
- Can we bring the design to the level of VLSI design?