One hundred years separate us from the time that I. A. Vyshnegradskii's work on the theory of direct (1877) and indirect (1878) regulation came to light. The problem of the dynamics of automatic regulation of steam engines was first stated and solved in these papers. The ideas and methods in Vyshnegradskii's work stimulated those of L. Lecornu, A. Stodola, N. E. Zhukovskii (Joukowski), M. Tolle, and other authors. As a result, at the end of the past century and the beginning of the present one the theoretical foundations were laid on which the edifice of contemporary control theory was raised from the thirties through the fifties. Vyshnegradskii can rightfully be considered the founder.

To mark the 100th anniversary of the day of publication of Vyshnegradskii's paper, the Editorial Staff publishes (in slightly abridged form) the report read by by Academician A. A. Andronov in January, 1949, at a session of the Engineering Sciences Branch of the Academy of Sciences of the USSR.

The Editorial Staff

I. A. VYSHNEGRADSKII AND HIS ROLE IN THE CREATION OF AUTOMATIC CONTROL THEORY (100 YEARS FROM THE DAY OF PUBLICATION OF I. A. VYSHNEGRADSKII'S WORK ON AUTOMATIC CONTROL THEORY)

A. A. Andronov

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§ 1. Introduction

The works of Ivan Alekseevich Vyshnegradskii (1831-1895) can be studied from a number of aspects, of course, mutually related.

Firstly, we can look upon Vyshnegradskii as an outstanding scientist, the founder of automatic control theory.

Vyshnegradskii's famous paper "On direct-action governors" [1] answered vitally important questions on the practice of constructing governors and marked an epoch in regulation theory. There is no doubt about his fundamental influence both on the practice of governor construction as well as on the subsequent development of regulation theory. Vyshnegradskii's other paper "On indirect-action governors" [2], devoted to certain nonlinear problems in regulation theory, also had an essential significance.

*The publication was prepared by M. A. Aizerman.

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Secondly, we can study Vyshnegradskii's activities as a prominent professor, the creator of the first Russian school of scientifically grounded machine-building engineers, and one of the most distinguished organizers of higher technical education in Russia. In the Russia of the nineteenth century, there were two lines in the matter of education and two scientific schools in the area of machine building. One of these lines was connected with Vyshnegradskii and with the St. Petersburg Technical Institute; the other line, the other school, somewhat later in starting its activity, was connected with the Moscow Higher Engineering School and with F. E. Orlov and N. E. Zhukovskii (Joukowski).

Thirdly, Vyshnegradskii, although a mathematician by training, was an outstanding practical engineer by whose designs and plans there were built and reequipped dozens of large-scale industrial enterprises. A considerable part of these enterprises were cartridge, gunpowder, small-arms, and cannon factories. Vyshnegradskii's role in reforms in the Russian artillery business, started in 1863, was very significant. It was Vyshnegradskii who in 1862-1878 virtually laid down the task of building and reequipping artillery works and organizing the mass production of artillery ammunition.

Fourthly, Vyshnegradskii was a prominent popularizer and propagandist in the area of exact natural science and technology. His lectures on mechanics, his famous published popular lectures on machines and his lectures on the basic laws of the mechanical theory of heat attracted large audiences and, published later, were widely used. Not only did Vyshnegradskii himself spend time on reading many published lectures, but he attracted his own students and colleagues to this matter of science popularization and propaganda.

Fifthly, Vyshnegradskii's unusual biography is of undoubted interest. The son of a provincial priest, appearing in St. Petersburg almost on foot, without means or connections, a genuine and most indigent "raznochinets,"* certainly very talented and energetic, he stands as a famous professor, as the creator of a school in the areas of applied mechanics and machine building, as the author of remarkable papers laying down the foundations of regulation theory. This eminent scientist was then made a member of the board of directors of a number of railroads and other industrial enterprises, a shareholder, a stock-exchange member and banker, and, finally, when he had almost given up his science and engineering studies, he was appointed as the Minister of Finance of the Russian Empire.

Vyshnegradskii's biography is of interest also because he was connected with many prominent people of his time. In particular, he knew D. I. Mendeleev very well, to whom, thanks to K. A. Timiryazev, he gave "refuge" in the Finance Ministry after the great chemist was forced to leave St. Petersburg University; he attracted Mendeleev to the elaboration of customs-tariff and entrusted him with the organization of the Board of Weights and Measures for the Finance Ministry; Vyshnegradskii had friendly relations with N. P. Petrov, the "father of the hydrodynamic theory of friction," with A. V. Gadolin, the outstanding specialist on artillery technology and well-known crystallographer, and with many other scientists.

It is self-evident that in the present report I could throw light only on Vyshnegradskii's role in the creation of automatic control theory, which is frequently misrepresented and ignored. It seems to me that in this respect, particularly now when automatic regulation and control have acquired a vast national-economic and defense value, we should bring in complete lucidity. Vyshnegradskii's activities as a stock-exchange member, and then as a Minister of Finance, cannot overshadow either his most important scientific results in the area of automatic control theory or his merit as the head of the first Russian school, originating in his own activities as far back as the 1860's in the areas of applied mechanics and machine building.

Entering upon the exposition of Vyshnegradskii's merits, I wish to note that many aspects of today's report were composed during my conversations with the late Corresponding Member of the Academy of Sciences of the USSR, Ivan Nikolaevich Voznesenskii.

§ 2. The State of Regulation Theory up to the Appearance of Vyshnegradskii's Paper

At first glance it seems that the principle of action of the usual centrifugal governor is rather obvious, that a knowledge of mechanics leads rapidly to carrying out all the necessary engineering calculations. However, matters proved to be not so simple. When in the second half of the last century the practice of the just-emerging science of machine building called urgently for an answer to the question of how to design the governor, how

*There is no English equivalent for this word. It means "an intellectual not belonging to the gentry in nineteenth-century Russia " – Translator.

to predetermine theoretically its construction data so that it worked without failure, this caused a great deal of difficulty. The urgency and vitalness of the problem occasioned the appearance of a large number of theoretical papers, but up to Vyshnegradskii these papers were almost never put to practice.

I shall not go into the details of the papers in which governors were analyzed statically. In these papers the engine's angular velocity was taken as specified and the governor was analyzed essentially as a tachometer whose configuration depends upon the given angular velocity. Such configurations could be stable or unstable. It was reckoned that only the governors stable in this static sense could be used in practice. In particular, the so-called astatic tachometers were strived for, they being considered as most precise; these are tachometers which could be found in equilibrium only at one specific angular velocity. The simplest example of such a tachometer is the so-called parabolic tachometer in which the balls of a centrifugal pendulum move along a parabola. It was assumed that the application of such astatic governors ensured the maintenance of the specified angular velocity of the machine under various loads.

These vague reasonings by themselves did not have a real mechanical justification. As is turned out later, these static stability conditions were neither necessary nor sufficient conditions for stable regulation.

We shall not dwell in detail on these static investigations and will pass on to the papers which examined the dynamics of regulation, i.e., the dynamics of the system consisting of two objects, viz., the machine and the governor between which there is a dynamic interaction. In a theoretical analysis such a dynamic interaction requires the setting up of the equations of motion and their integration, exact or approximate. Thus, the dynamic problem that emerged was far from simple and quite a long time was needed from the moment the problem was posed before a real progress was achieved.

The first paper on regulation dynamics was published by Airy in 1840 [3]. It came about because of the demands of practice in astronomic investigations. Astronomy had already for a long time called for special friction clockwork mechanisms which would enable the equatorial to observe automatically the apparent motion of a star.

Such a friction clockwork mechanism is essentially a certain friction governor. In the simplest case this mechanism consists of a centrifugal pendulum whose arms move outwards as the angular velocity increases and the weights rub against the inner surface of the clockwork mechanism's circular housing. As the angular velocity grows the friction force moment generated by the balls increases, and as the angular velocity diminishes the friction force moment decreases, and thus the angular rotation velocity is regulated. Although such a friction governor does not give the accuracy required, for example, in the usual clocks, it nevertheless creates a uniformity of equatorial rotation sufficient for the usual astronomic purposes.

Airy observed that very often the working of such clockwork mechanisms is accompanied by undesirable oscillations. He set himself the goal of making a theoretical analysis of this question and of finding with the aid of the theory a means of combatting these phenomena. Having made certain assumptions on the dependence of the friction force moment on the angle of opening of the balls of the centrifugal pendulum and not paying attention to the friction in the collect and the articulations of the governor itself, he set up the equations of motion and integrated them. Airy did not know what we now call the theory of small oscillations around the motion's steady-state equilibrium; therefore, his calculations were exceedingly cumbersome and call to mind the calculations in the astronomic theory of perturbations.

From his own theory, disregarding the friction in the governor articulation, Airy concluded that the governors being investigated by him always operate unstably. This pessimistic conclusion forced him to think that some essential change in the construction was necessary.

Subsequently Airy fit to the clockwork mechanism the so-called dashpot, a singular device, which is attached to the governor's clutch and creates a friction proportional to the rate of displacement of the governor's clutch; the coefficient of proportionality can be changed to the constructor's desire. With the aid of such a dashpot he succeeded in constructing governors which worked without harmful oscillations. However, Airy did not give the theory of a governor with a dashpot, in particular, the operating stability conditions of such a system. Not knowing the theory of small oscillations around the steady-state motion, this was not so simple to do.

The next paper in regulation theory, having a considerably greater significance, was that by the famous English physicist James Clark Maxwell, "On governors," published in 1868 [4]. The appearance of this paper also was connected with a certain distinctive concrete problem. Maxwell's collaborator, Engineer Fleeming Jenkin, invented a singular type of governor that differed essentially both from the usual centrifugal governor as well as from that friction governor of which we just spoke. The governor consisted of a centrifugal pendulum whose balls, diverging, touched the inside surface of a special ring which could rotate around the axis of symmetry. The ring was found under the action of a constant moment generated by a special weight and was connected by a system of levers with a damper. In concept such a governor could maintain constant the number of revolutions of the engine in spite of load changes. In modern terminology this governor can be called an astatic regulator of special construction. Jenkin's governor did not always work well. During its operation undesirable oscillations frequently arose.

Maxwell became interested in the stability conditions for the working of Jenkin's governor as well as for other analogous astatic governors. He understood that the stability conditions for the working of systems can be obtained comparatively simply by making use of the theory of small oscillations around the steady-state motion. The thing is that Maxwell was previously occupied with questions close to this one from the point of view of theoretical mechanics in his important paper on the stability of Saturn's rings. In the paper "On governors" Maxwell linearizes the problems he is considering on the motion stability of engines equipped with different astatic governors and reduces the stability question to an algebraic problem, viz., to the investigation of the roots of the so-called characteristic equation. According to Maxwell the steady-state motion of the engine–governor system will be stable if all the roots of this characteristic equation have negative real parts. Here Maxwell states that the phenomenon of self-oscillations in the engine–governor system, arising as some parameter changes, is connected with the transition from a stability domain wherein all roots of the characteristic equation have negative real parts into an instability domain wherein at least one of the roots has a positive real part. Maxwell was in a position to find such conditions only for third-degree equations. In the same year 1868, at the London Mathematical Society, Maxwell asked the question, entered into the minutes, of whether any of the Society's members could indicate such conditions for equations of any degree or indicate a method by which such conditions can be found.

After several years the problem that Maxwell posed was solved by E. J. Routh for fourth- and fifth-degree equations. In 1877 the same Routh in the monograph "Stability of a Given State of Motion,"* written at the suggestion of Maxwell and Stokes, solved this problem in general form, or, more precisely, gave an algorithm by which the problem can be solved successively for a characteristic equation of any degree.

Maxwell's paper was an essential step forward in the area of theoretical mechanics. In many respects this paper is of significant interest. A simple comparison of the papers by Airy and by Maxwell shows what a large step the transition to the motion-stability analysis of the engine–governor system by the method of small oscillations was in the matter of creating a theory of regulation.

But the theory of machine regulation, answering to the questions of industrial practice, did not begin with Maxwell. Maxwell was a physicist and not an engineer. This manifests itself first of all by the choice of the governor systems with whose theory he started to occupy himself. Jenkin's invention, of which we spoke, was certainly very clever, but the maintenance with high accuracy of the number of revolutions was by no means a vital problem of industrial technology in Maxwell's time. This invention was of interest to physicists and astronomers. For instance, Lord Kelvin devoted his singular article, in truth, not to the giving of a real theory of its action. But this invention did not meet with much enthusiasm among engineers and after being put to work on several experimental installations it was forgotten. The other types of astatic governors with which Maxwell busied himself met with roughly the same fate.

It should be particularly noted that Maxwell passed by Watt's ordinary centrifugal governor. He even denied it the name of "governor," referring it to the so-called moderators. This circumstance, i.e., the fact that he did not take interest in engineering governors, significantly depreciated his paper in the eyes of engineers. But, perhaps, something else is material; Maxwell, by applying the theory of small oscillations to the motion of the engine–governor system, did not clearly realize the relative value of the coefficients of the corresponding systems of linear differential equations of motion. We can say this differently: Maxwell was not aware of precisely how the physical factors in practice caused the stable operation of a steam engine equipped with Watt's ordinary centrifugal governor. Therefore, the recommendations given to engineers by Maxwell in his theory, and the main recommendation, viz., the recommendation of an astatic governor, was automatically improper for that time. For machines with a small self-oscillation–and it was precisely such machines in those years–astatic governors are unfit. This has been subsequently well proven.

Neither in Maxwell's deductions nor even in his equations could the engineers find clear indications of how to combat the tendency, manifesting itself in many cases, of steam engines equipped with the ordinary Watt governors to unstable operation and to hunting.

*Adams Prize Essay, Macmillian and Co., Ltd., London (1877)–Translator.

§3. Vyshnegradskii's Work on Direct-Action Governors and Its Value in Governor Construction Practice

A control theory for machines, answering the demands of industrial practice, was first given in Vyshnegradskii's 1876 paper "On direct-action governors" [1]. Although a mathematician by training, Vyshnegradskii was a practical engineer. He set himself the concrete problem of analyzing the motion of an ordinary steam engine equipped with an ordinary centrifugal governor.. Maxwell looked upon the problem of control theory as a problem of theoretical mechanics. To Vyshnegradskii, regulation theory was the main road to the design and construction of steam engines. In the long run Vyshnegradskii's theoretical analysis led to a compact calculation formula and to an elegant graph permitting the rapid utilization of the theoretical results in practice.

Vyshnegradskii's fundamental merit in regulation theory is that he was correctly able to apply the theory of small oscillations to the engine–governor interaction problem. Particularly remarkable was the problem's schematization adapted to the essence of the question. He accounted for that which must be accounted for and discarded everything that superfluously complicated the problem, which would make the understanding of the dynamics of the control process difficult. Vyshnegradskii accounted for the presence of the dashpot generating the viscous friction. He could not get away with neglecting this since if he were to ignore the viscous friction he would obtain instability. In his final scheme, connected with the theory of small oscillations, he could not account for Coulomb friction since then the problem is nonlinear and the solution becomes very complex. However, Coulomb friction did figure in some of Vyshnegradskii's initial formulas, which he deemed necessary to introduce the concept of "governor sensitivity."

It is very essential and nontrivial that Vyshnegradskii right away relinquished the accounting for the engine's self-regulation, since the usual industrial steam engines of his time possessed little self-regulation, not being of a practical value.

It was far from simple to carry out such a proper schematization. Maxwell allowed for viscous friction, but along with it he also allowed for self-regulation and discarded nonuniformity, and obtained deductions having no practical value.

Worms de Romilly [5] examined the same problem as did Vyshnegradskii but he did not take friction into account and arrived at the paradoxical deduction that all governors are unstable.

Kargl [6] analyzed a steam-engine control problem in which the governor controls the steam chamber and arrived at a considerably more complex mathematical problem which he was unable to solve up to the end.

We can say that Vyshnegradskii was the first to properly schematize the problem of the motion of the ordinary steam engine equipped with the ordinary Watt centrifugal governor and to propose the beginnings of a regulation theory capable of being a material aid to governor construction.

Vyshnegradskii examined a particularly simplified load variation scheme. He reckoned that a load variation by a specific finite amount occurs instantaneously and analyzed the behavior of the engine–governor system after such an instantaneous variation, assuming that subsequently the load now does not undergo any variations and that the engine operates at a constant load, interacting with the governor. Using the theory of small oscillations he reduced the stability investigation to that of a third-degree equation.

He expressed the theory's final results by a compact formula and by an original and clever graph. The formula and the graph permit a rapid utilization in practice of the results of the theoretical investigation. The graph, or the Vyshnegradskii diagram as it is called, gives an intuitive picture of how the system's stability changes as a function of the constructive parameters of machine and governor. Since Vyshnegradskii applied the theory of small oscillations and linearized the equation of motion, for him the stability of regulation is independent of the magnitude of the load drop.

From the formula or from the diagram there directly follow a number of very important statements:

1) The governor's equivalent masses adversely affect the stability;

2) the governor's operation cannot be stable without friction;

3) a specific nonuniformity of the governor is necessary, i.e., it is necessary that a load variation lessen somewhat the steady-state angular velocity;

4) the moment of inertia has a positive influence on stability, i.e., the greater the moment of inertia, the farther we are from instability.

In order to make his deductions palatable to engineers and to attract attention to the more important ones, Vyshnegradskii formulated his famous thesis at the end of the paper.

These deductions of Vyshnegradskii created an epoch in control theory. The engine–governor–system dynamics became clear and it became understood to which side one must vary some constructive parameters or others in order to increase the system's stability.

In order to clarify precisely what Vyshnegradskii did here, it is necessary to compare his paper with that of Maxwell on governors, of which we have already spoken. I present my own conversation on this matter with Voznesenskii, Corresponding Member of the Academy of Sciences of the USSR, the prominent Soviet engineer, a specialist in the areas of hydraulics and regulation, who passed away in 1946. When I brought him a photocopy of Maxwell's 1868 paper which presented the linear equations of a number of astatic governors and gave the conditions for their stable operation, and wherein Maxwell recommended an astatic governor to engineers, Ivan Nikolaevich exclaimed: "Here is an astonishing thing. Essentially, with the aid of one and the same theory of small oscillations, using one and the same condition of the negativity of the real parts of the roots of a cubic equation, Maxwell, from the point of view of practice, muddled up the question, while Vyshnegradskii untangled it." He said, further: "Of course, Maxwell's paper is very interesting. Relying on it one can study in general form the theory of the interaction of engine and governor. But only after Vyshnegradskii could engineers understand what they had to do in order to have stable control." Perhaps, the word "muddled" should be softened, but if we ask the simple question of who was the first to give the theory of operation of an engine equipped with the ordinary Watt centrifugal governor, the answer is: Vyshnegradskii gave this theory.

It should be stressed – we shall not dwell on this – that Vyshnegradskii gave not only a theory for calculating a governor, in particular, its stability, but, as far as I know, to him is due the first investigation of transient performance. There is nothing on this investigation in the Russian version of the paper on direct-action governors, but it comprises Sec. 14 of the German version [7].

Without dwelling in detail on this supplement, we merely observe that in it Vyshnegradskii clearly indicates that if we wish to have a governor with small nonuniformity, then we must not strive for all the roots of the cubic equation to be negative and real; otherwise, it is very difficult, practically impossible, to realize a governor which would have a small nonuniformity and for which the transient response would be monotone. It is interesting to note that Vyshnegradskii understood that there can be a monotone response in the case of complex roots, and, conversely, understood that even if all three roots of the characteristic equation are real and negative, there can nevertheless be some oscillations under specific conditions. Undoubtedly, in this matter, i.e., in the question of the theoretical analysis of the transient performance and in the understanding of the dynamics of such a response, Vyshnegradskii was significantly ahead of his time.

Let us go on to evaluate the state-of-the-art of regulation theory up to the time of appearance of Vyshnegradskii's paper. The centrifugal governors placed on steam engines at the end of the eighteenth century worked fairly. Such fair and failure-free operation of governors continued up to the middle of the nineteenth century. But, as the capacity of the steam engines increased, as the engines became faster, and as the masses of the governors grew, the operation of the governors came to be far more often accompanied by deleterious oscillations, by loosening. Knowing the results in Vyshnegradskii's paper, we can easily analyze what the matter is. According to Vyshnegradskii, friction is necessary for the stable operation of a governor. But the performance of governors and of the articulations and the handling of friction in the parts were considerably improved by the end of the 1850s.

Thus, friction became considerably less. On the other hand, the governor masses increased as the need appeared of moving the dampers on engines possessing significantly large capacity, while in connection with the increasing number of revolutions of the engines, the flywheels' moments of inertia became less. In addition, since the engineers did not know that a small nonuniformity has an adverse influence on stability, while they wished that there was less change possible in the number of revolutions as the load varied, governors came to be made with a very small nonuniformity. All of these caused harmful effects on the stability, as is not difficult to perceive from Vyshnegradskii's formula or diagram. Hence the reason for failure in governor operation is clear.

In view of the fact that governor instability was detected rather frequently, the designers up to the appearance of Vyshnegradskii's paper started to think of changes in the very reasoning behind the construction of Watt's governor. They believed that a governor that starts to deflect only after a change in the angular velocity is unsatisfactory in principle, that it was necessary to regulate such that the governor started to deflect at the instant of appearance of angular acceleration at which the angular velocity still has not had time to change.

Other ideas on how to construct governors differing from Watt's were expressed. However, the governors constructed along these new lines also often worked unsatisfactorily. Vyshnegradskii's paper appeared at just the critical moment in governor construction and, therefore, its value was very great.

During the few years after Vyshnegradskii's paper, as its results spread, engineers came to understand the direction in which the parameters had to be changed in order to obtain reliable and stable governor operation. They came clearly to see the ill effects of massive governors on stability and the useful influence of the moment of inertia and of the nonuniformity on regulation.

It was considerably more difficult to comprehend the thesis on the dashpot, to which Vyshnegradskii attached a sharp form. Engineers knew very well that reliable operation governors existed which were not equipped with any special dashpots generating viscous friction. However, gradually, they made out that viscous friction, or, what is the same, the dashpot, can in practice be replaced by the usual Coulomb friction that always exists in the governor's articulations. Vyshnegradskii's thesis that "without a dashpot there is no governor" became understood as the thesis that "without friction there is no governor" and it was considered that to obtain qualitative results there is need to bring in, instead of the Coulomb friction in the equations of motion, some equivalent viscous friction.

§4. Acknowledgement of Vyshnegradskii's Merit in the Creation of Control Theory

We pass on to the question of acknowledging Vyshnegradskii's merit in the creation of control theory. Here we observe much that is paradoxical. A. I. Sidorov in the book "Horizontal Governors" (1892) displayed a complete lack of understanding of Vyshnegradskii's paper and gave it an improper evaluation.

Joukowski in the book "Control Theory of Engines" (1909), the most widely distributed book on control theory in Russia, containing a remarkable exposition on the classical theory of direct-action governors and some essential new results, indicated that Vyshnegradskii has made an error in his investigations in not taking into account that he was dealing with an equation possessing a discontinuous right-hand side and that he would need to choose new constants of integration in each range.

V. D. Kirpichev, in spite of the fact that he was Vyshnegradskii's student and had great respect for him, remarked in "Discussions on Mechanics" that in the first papers on regulation (and it is self-evident here that he had Vyshnegradskii's papers in mind) the Coulomb friction was improperly differentiated since the authors forgot about its discontinuous nature and, therefore, obtained improper results.

Vyshnegradskii's name is not mentioned at all in another well-known Russian book on engine control theory, that by E. L. Nikolai.

In a number of foreign books too we encounter a negative evaluation of Vyshnegradskii's work and a mention of his error. The question is, what is happening here? Where did such an attitude to this remarkable paper originate?

It seems that the definitive role here was played by Sidorov who, not only in his book "Horizontal Governors" (1892) but also in a number of other books, speaking of Vyshnegradskii's paper, mentioned the error in it. But the principal role in this negative attitude was played by a book by the French engineer Lecornu [8], since the majority of authors (including Joukowski) refer to Lecornu when mentioning Vyshnegradskii's error.

This underestimation of Vyshnegradskii's paper could not prevent the spreading of its merit both in Russia and abroad. The singular role of A. Stodola in this matter will be noted below. It is difficult to list all the books and articles which speak of Vyshnegradskii's merit in the creation of regulation theory. I mention only a few. In the book by the German engineer Hort, "Engineering Theory of Vibrations," and in his article devoted to the history of regulation theory and to intermittent regulation, it is directly stated that Vyshnegradskii's paper on direct-action governors, in particular, his famous theses, is the foundation of modern regulation theory. In the book by the German engineer Lorentz, "Engineering Mechanics," widely distributed and translated into Russian in its own time, it was written that the theory of motion of an engine equipped with a Watt governor was first given by Vyshnegradskii.

Among the articles published in the USSR we should particularly note the article by the late Corresponding Member of the Academy of Sciences of the USSR, I. N. Voznesenskii. Vyshnegradskii's role in the creation of regulation theory is commonly acknowledged at the present time in the USSR thanks to Voznesenskii to a great extent.

Abroad the role of founders of control theory is not infrequently conferred to persons (Tolle, Trinks, etc.) who are compilers and pedagogues, but in no way are the creators of this theory. In this regard we should take special note of the book by the American Trinks, published in New York in 1919, in which we can read that Vyshnegradskii's paper is the foundation of modern regulation theory. Trinks noted that this paper had not received the fame that it deserved; he himself evaluated it very highly.

What exactly is this business about Vyshnegradskii's error? From Vyshnegradskii's article and from a comparison of it with the German version of this same article and with a note on it in the Reports of the French Academy of Sciences it is perfectly clear that there is no mathematical error whatsoever in Vyshnegradskii's work and that nowhere did he differentiate a discontinuous term. In the Russian version of the paper it is stated quite definitely that the Coulomb friction is analyzed provisionally with the idea that it is subsequently neglected. In the German version the corresponding expressions are written out without the Coulomb friction. In addition, it is clearly mentioned, particularly in the note in the Reports of the French Academy of Sciences, that governors can operate stably in the absence of a dashpot. From where then is Vyshnegradskii's thesis on the dashpot?

The origin of this thesis is not connected with any mathematical error. As we have remarked, Vyshnegradskii did not admit the presence of any appreciable insensitivity in the governor. Therefore, he considered it improper to use Coulomb friction for the stabilization of the motion of the engine–governor system and assumed that it was necessary to decrease maximally the Coulomb friction and to achieve stability by using the dashpot.

From the contemporary point of view Vyshnegradskii's conclusion is not entirely proper since in many cases we can achieve a stable control process with a Coulomb friction which creates insensitivity, completely tolerable in practice. But this is no way mars Vyshnegradskii's paper: Then, that Vyshnegradskii was the first to clarify the very mechanism of the interaction of the governor with the engine, that he ascertained the role of mass, the role of friction, the role of nonuniformity, all that remains in full force.

§5. Development of Control Theory after Vyshnegradskii

We now endeavor to characterize briefly the development of regulation theory after Vyshnegradskii. Together with the usual governors, the so-called indirect-action governors had already gained prevalence by the time Vyshnegradskii's paper appeared. In these governors the variation of the dampers controlling the access of steam in the engine was not accomplished directly by the sensing element. Here the sensing element accomplishes only the engaging and disengaging of the mechanism moving the dampers, with the main shaft of the engine. The energy for displacing the dampers is provided by the engine's shaft. Such a system had a discontinuous action. It was impossible to linearize such a system for the convenience of a theoretical analysis; we had to solve in essence a complex nonlinear problem.

Vyshnegradskii occupied himself with these nonlinear problems in his second paper "On indirect-action governors" [2], which is of considerable theoretical interest. In substance this was the first paper in the world of science on a nonlinear theory of regulation; however, its value is in no way comparable with the results of the first paper on direct-action governors. The fact is that by the time this paper of Vyshnegradskii appeared, the indirect-action governors with whose theory he busied himself had already gone out of demand. Indirect-action governors based on another principle had appeared. In these new governors the sensing element moved the dampers of a singular hydraulic or oil motor whose piston was connected with the damper. The hydraulic motor had the so-called rigid feedback which permitted the coordination of the position of the hydraulic motor's piston and the position of the governor's clutch.

A hydraulic motor with rigid feedback was named a servomotor. The introduction of such servomotors signified an essential progress in the practice of governor building. This had a special value in the regulation of water turbines where the moving of the dampers called for large forces and where the introduction of indirect-action governors of the new type gave the possibility of solving the whole problem more effectively.

Following Vyshnegradskii, Aurel Stodola (1859-1942), engineer and professor at the Zurich Polytechnicum, linearized the problem of indirect control. Stodola's activities, just as those of Vyshnegradskii, can be studied from a number of aspects.

Stodola is Vyshnegradskii's successor in control theory, obtaining fundamental scientific results along the path pointed out by Vyshnegradskii.

Stodola is well versed in the area of design and calculation of heat engines and of the design of steam and gas turbines, a professor at a higher technical school in the area of machine building and heat engines, of the design and erection of a number of hydraulic and heat installations. Stodola had to encounter the problems of controlling water turbines. Indirect regulation dominated this area. Hydraulic servomotors with rigid feedbacks, lending themselves to linearization, were widely prevalent. The system with which Stodola was bound to be busy consisted of a governor, a servomotor, turboconductors, an air box, and a water turbine.

In his own reasonings and calculations, Stodola [9, 10] is a direct successor of Vyshnegradskii. He investigated the simplest cases, which he succeeded in reducing to a third-order equation, in the manner of Vyshnegradskii, citing him. But in a number of cases, when Stodola arrived at a characteristic equation of fifth order or higher, he turned for help to the mathematician Hurwitz, his friend at the Zurich Polytechnic Institute. Stodola and Hurwitz knew neither of Maxwell's papers nor of Routh's papers. Hurwitz solved all over again the problem that Maxwell had posed roughly 25 years before this and that Routh had solved 15 years before this. But Hurwitz did not simply repeat Routh's solution. More precisely, Hurwitz, using one of the papers by the French mathematician Hermite, succeeded in obtaining a condition for the negativeness of the real parts of an n-th-degree algebraic equation in a closed form, in the form of a series of inequalities written in an elegant determinant form. This criterion of Hurwitz, although it differs in form from Routh's criterion, coincides with it in essence. At the present time these inequalities bear the name Routh – Hurwitz inequalities, but this address to Hurwitz touches only the mathematical aspect of the question. Regarding the mechanical and engineering ideas in the area of regulation, Stodola follows Vyshnegradskii, and we speak specifically of this. In particular, in the second part of his paper, where Stodola examines the influence of the governor's mass and of the dashpot's action and where in mathematical respect he follows Hurwitz, he declares that Vyshnegradskii's research determined in the main the course of his own reasonings.

This paper by Stodola is remarkable in other respects. In it he introduced for the first time in the differential equations of the regulation system the so-called time constants which have been retained in it up to the very present. Some years after this paper Stodola once again turned to regulation theory and gave a theory of horizontal inertial governors [10]. Thus are named governors in which besides the usual centrifugal force of inertia there further act tangential forces of inertia. Here too he follows Vyshnegradskii. He shows how Vyshnegradskii's famous thesis must be altered, referring to the usual direct-action governor, for the case of an inertial governor. In particular, he shows that the famous thesis "without nonuniformity there is no governor," valid for the usual centrifugal governor, now has no force here. In this paper Stodola dwells in detail on clarifying the role of Coulomb friction. He makes approximate calculations for the cases of the nonlinear problem, when viscous friction is absent and only one Coulomb friction obtains; he shows that such friction can stabilize the engine – governor system and gives an appropriate quantitative estimate. He elucidates in detail the significance of Vyshnegradskii's theses and takes these theses under his wing. In particular, he somewhat modifies Vyshnegradskii's thesis "without a dashpot there is no governor" and replaces it with his own thesis "without friction there is no governor."

These papers by Stodola are a direct continuation and development of Vyshnegradskii's paper and complete his own classical theory of the regulation of motors. The main line of development of regulation theory is: Vyshnegradskii – Stodola – Tolle; a collateral line is: Maxwell – Routh. In Stodola's latter paper these lines merge since in it there is reference to Routh's book and it contains an indication on how to use the theory of small oscillations for calculating any arbitrarily complex regulation system. In Tolle's book [11], published later and serving as the basic handbook on regulation theory in many countries of Europe, particularly Germany, there is no reference to Vyshnegradskii and there is only some mention of Stodola's papers. But Tolle's book should be treated as being of a compilatory and pedagogical nature. As a matter of fact Tolle merely set forth, very clearly and intelligibly it is true, Vyshnegradskii's paper, Stodola's papers, and his own comparatively insignificant new scientific results.

It is not the task of the present report to analyze the papers in the area of regulation theory written in the last decades. We therefore limit ourselves to brief remarks. The development of regulation theory can be separated into three stages. The first stage, concluding at the beginning of the twentieth century and summed up in Tolle's book, is the theory of the automatic regulation of motors (steam engines, water turbines, steam turbines). The coordinate being regulated here was the angular velocity of the motor's shaft. The second stage is connected with the extension of the classical theory to the regulation of other quantities: electric voltage, temperature, pressure, consumption, etc., since the regulation of these quantities has obtained vast significance. To this second stage of development of the theory we must refer the use of linear equations for seeking the stability conditions for the operation of the ordinary autopilots and other comparatively simple devices of automatic control. We can reckon that the second stage ended with the start of the Great Patriotic War. The third stage, the contemporary one, is characterized by a process of fusion of the linearized theory of regulation and control with the linearized theory of multiunit and multiloop amplifiers. This fusion is particularly noticeable and fruitful in servomechanism theory wherein it has gone a very long way.* The third stage is characterized by the great extension of various geometric stability criteria for the linearized systems. Of most interest here are the Nyquist diagram (1932), the Mikhailov diagram (1938), the Neimark diagram (1947), enabling us in many cases to answer rapidly and visually the questions of stability. All these diagrams can be looked upon as the natural complex generalizations of the ordinary real Vyshnegradskii diagram.

We see that the main channel along which the theory of regulation and control has developed until now is the linearized theory. In this sense we can say that it has developed in the direction whose practical effectiveness was proved in Vyshnegradskii's own paper.

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*Remember that this text was written in 1949. All succeeding development of control theory followed the path laid down by the papers of Vyshnegradskii and Stodola. – The Editorial Staff.