Introduction to Time-Delay Systems

Fall 2012

Homework no. 2

(submission deadline: 7.11.2012, 10:00am)

Problem 1 (40%). Find all regions of $h > 0$ for which the quasi-polynomials below are stable:

1. $\chi_{h,1}(s) = s^2 + 1 + \mathrm{e}^{-sh}$
2. $\chi_{h,2}(s) = -s^2 + 1 + (2s^2 + 4s + 3)\mathrm{e}^{-sh}$
3. $\chi_{h,3}(s) = s^2 + 0.04s + 1 + 0.5(s^2 + 0.2s + 1)\mathrm{e}^{-sh}$
4. $\chi_{h,4}(s) = s^2 - 3s + 7 - 3\sqrt{5}\,\mathrm{e}^{-sh}$

Problem 2 (30%). Let $\chi_h(s) = P(s) + Q(s)\mathrm{e}^{-sh}$ for some $P(s)$ and $Q(s)$ satisfying $\boldsymbol{\mathcal{A}}_{\mathbf{1\text{–}4}}$ from the lecture notes. Prove that if $P(0) + Q(0) \le 0$, then $\chi_h(s)$ is stable for none $h > 0$.
(Hint: remember Vièta's formulas)

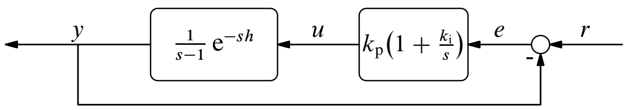

Figure 1: PI stabilization of an unstable time-delay plant

Problem 3 (30%). Consider the feedback system depicted in Fig. 1, where an unstable plant with delay is controlled by a PI controller.

1. What is the range of delay h for which the closed-loop system is stable ? Derive it as a function of the controller coefficients and plot stability contours in the k_p–k_i plane for $h \in \{0.1, 0.2, 0.3, 0.5, 0.7\}$ and with $k_\mathrm{p} \in [0, 10/3]$ and $k_\mathrm{i} \in [0, 1.6]$.
2. What is the maximal admissible delay h for which the closed-loop system can be stabilized by a PI controller ?
3. What is the maximal admissible delay h for which the velocity gain of the feedback loop $|k_\mathrm{p}k_\mathrm{i}| \ge \frac{1}{2}$ can be attained ?