LP feasibility via ADMM. Implement an ADMM method for finding $x \in \mathbf{R}^n$ that satisfies $Ax = b$ and $x \succeq 0$, where $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$ are given data, with $m < n$ and A full rank. Use factorization caching to reduce the computational effort per iteration, after the first one. To check convergence, you can plot two residuals versus iterations. One quantity in your algorithm will be nonnegative, but satisfies $Ax = b$ only in the limit. For this quantity you can plot the norm of the equality constraint residual. Another quantity in your algorithm will satisfy the linear equality constraint exactly (at least, up to numerical precision), and satisfies nonnegativity only in the limit. For this quantity, you can plot the sum of maximum of its negative entries versus iteration.

To make sure the data you use is feasible, you can generate the data as follows. First, generate A randomly (or any other way). Then take $b = Aw$, where w is any nonnegative vector (for example, uniform random or a multiple of $\mathbf{1}$).

When $Ax = b$, $x \succeq 0$ is not feasible, the dual variable in ADMM will diverge. Generate such a data pair to verify this.

You can use CVX to verify feasibility or infeasibility, as a second check.