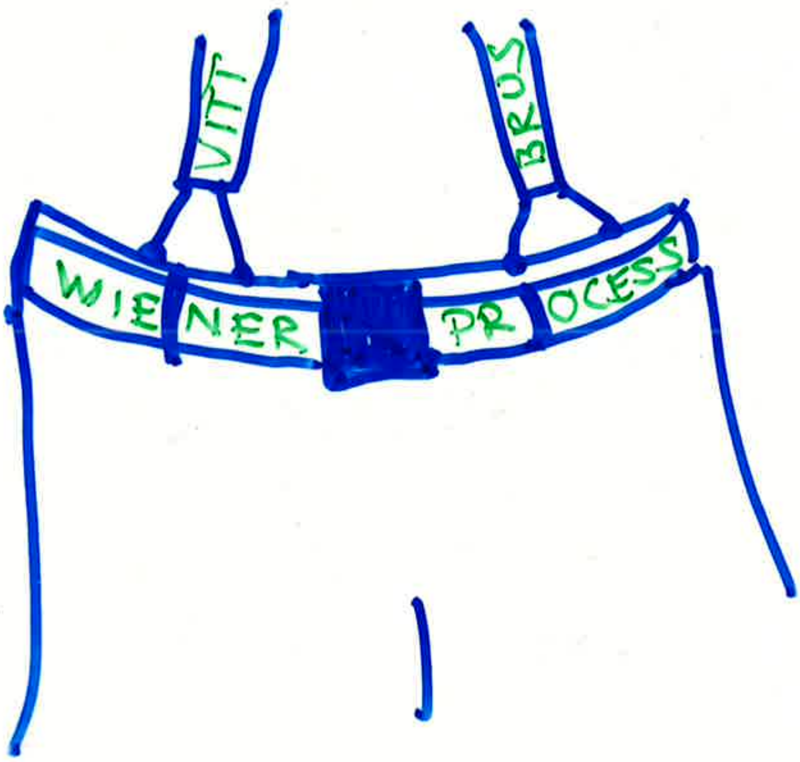
Dagens idé

Behövs både hängslen och livrem?



F2:

STOKASTISKA TILLSTÅNDS-MODELLER

- Tidsdiskreta system
- Stok. differensekvationer
- Problem med tidskont. stok. system
- Stokastiska integraler
- Stokastiska differential ekv
- Ito derivering
- Sampling

WIENER PROCESS

- Oberoende, stationära inkr.
- $\mathrm{Var}\, w(t) = c \cdot t$
- $r(s,t) = c \cdot \min(s,t)$
- w kontinuerlig, men ej deriverbar i medelkvadrat mening

VITT BRUS

- $r(s,t) = 2\pi c\, \delta(s-t)$
- $\phi(\omega) = \text{konst} = c$
- $e = \frac{dw}{dt}$
- Inga problem efter en integration

MARKOV PROCESS

$\{x(t),\ t \in T\}$ stok process

$t_1 < t_2 < \dots < t_k < t$

$$P\{x(t) \leq \xi \mid x(t_1), x(t_2), \dots, x(t_k)\}$$

$$= P\{x(t) \leq \xi \mid x(t_k)\}$$

$$F(\xi_1; t_1) = P\{x(t_1) \leq \xi_1\}$$

$$F(\xi_t; t) = P\{x(t) \leq \xi_t \mid x(s) = \xi_s\}$$

$$F(\xi_1, \xi_2 \dots \xi_k; t_1 \dots t_k)$$

$$= F(\xi_k, t_k \mid \xi_{k-1}, t_{k-1}) \cdot \dots \cdot$$

$$F(\xi_2, t_2 \mid \xi_1, t_1)\, F(\xi_1, t_1)$$

Theorem 4.1

$v(t)$ tidskont andra ordn process

1. $v(t)$ oberoende av $v(s)$ $t \neq s$
2. $v(t)$ kont. i medelkvadratmening och ändlig varians
3. $E\,v(t) = 0$

$\Rightarrow E\,v^2(t) = 0$ i medelkvadr. mening

Bevis 1 sida ointressant

Konsekvens INTRESSANT

Ex 5.1

$$\int_0^t w(s)\,dw(s)$$

$$I_0 = \lim \sum w(t_i)[w(t_{i+1})-w(t_i)]$$

$$I_1 = \lim \sum w(t_{i+1})[w(t_{i+1})-w(t_i)]$$

$$I_1 - I_0 = \lim \sum [w(t_{i+1})-w(t_i)]^2 = t$$

$$I_\lambda = (1-\lambda)I_0 + \lambda I_1$$

I_0 : Ito integral

$I_{0.5}$: Stratonovich integral

För Ito gäller

$$E\int f\,dy = \int E\{f(t)\}\,dm(t)$$

$$\operatorname{cov}\left[\int f\,dy, \int g\,dy\right] = \int E\{f(t)g(t)\}\,dr(t)$$

Jämförelse

Antag w "vanlig" funktion

$$\int_0^t w(s)\,dw(s) = \frac{1}{2}\left(w(t)^2 - w^2(0)\right)$$

w - Wiener process

$$I_\lambda = \frac{1}{2}\left[w^2(t) - w^2(0)\right] + \left(\lambda - \frac{1}{2}\right)\cdot t$$

Likhet om $\lambda = 0.5$ (Stratonovich)

Stokastisk differential ekvation

$$dx = A(t)\,x\,dt + dv$$

$$\quad\quad\quad\quad\quad\quad\quad \llcorner \in W(R, dt)$$

$$x(t_0) \in N(m_0, R_0)$$

Medelvärde

$$\frac{d m_x}{dt} = A(t)\, m_x(t)$$

Kovarians

$$R(s,t) = \Phi(s,t)\, P(t) \qquad s \geq t$$

$$\frac{d}{dt} \Phi(t, t_0) = A(t)\, \Phi(t, t_0)$$

$$\frac{dP}{dt} = A\phi R_0 \phi^T + \phi R_0 \phi^T A^T$$
$$+ I R_1 I^T + A \int \phi R_1 \phi^T ds$$
$$+ \int \phi R_1 \phi^T ds\, A^T$$

$$= A \left[\phi R_0 \phi^T + \int \phi R_1 \phi^T ds \right]$$
$$+ \left[\phi R_0 \phi^T + \int \phi R_1 \phi^T ds \right] A^T + R_1$$

$$= AP + PA^T + R_1$$

Th 6.1

$$\frac{dm_x}{dt} = A(t)\, m_x \qquad m_x(t_0) = m_0$$

$$R(s,t) = \begin{cases} \phi(s,t) P(t) & s \geq t \\ P(s)\, \phi^T(s,t) & s \leq t \end{cases}$$

$$\frac{dP}{dt} = AP + PA^T + R_1 \qquad P(t_0) = R_0$$

P(t)

$$P(t) = E\, x(t)\, x^T(t)$$

$$= E\left\{\left[\phi(t,t_0)x(t_0) + \int_{t_0}^{t} \phi(t,s')\,dv(s')\right] \cdot \left[\phi(t,t_0)x(t_0) + \int \phi(t,s)\,dv(s)\right]^T\right\}$$

$$= \phi(t,t_0)\,P(t_0)\,\phi(t,t_0)^T$$

$$+ E\left[\int \phi(t,s)\,dv(s) \cdot \int \phi(t,s)\,dv(s)\right]$$

$$= \phi(t,t_0)\,P(t_0)\,\phi(t,t_0)^T + \int_{t_0}^{t} \phi(t,s)\,R_1(s)\,\phi^T(t,s)\,ds$$

Derivera

$$\frac{dP}{dt} = \left[\frac{d}{dt}\phi\right] R_0\, \phi^T + \phi\, R_0 \left[\frac{d}{dt}\phi\right]$$

$$+ \phi(t,t)\,R_1(t)\,\phi^T(t,t) + \int \left(\frac{d}{dt}\phi\right) R_1\, \phi^T\, ds$$

$$+ \int \phi\, R_1 \left(\frac{d}{dt}\phi\right) ds$$