5.2.1. $A(z) = z^2 + 0.4z + 0.1$

$B(z) = z^2 + 0.9z + 0.8$

$$I = \frac{1}{2\pi i} \oint \frac{B(z)\,B(z^{-1})}{A(z)\,A(z^{-1})} \frac{dz}{z}$$

			α				β
$\underline{1}$	0.4	0.1		1	0.9	0.8	
0.1	0.4	1	0.1	0.1	0.4	1	0.8
$\underline{0.99}$	0.36			0.92	0.58		
0.36	0.99		0.36364	0.36	0.99		0.58586
$\underline{0.85910}$			1	0.70909			0.82538

$$I_2 = \frac{1}{a_0^2} \sum_{i=0}^{2} \frac{(b_i^i)^2}{a_0^i} = \frac{(0.70909)^2}{0.85910} + \frac{(0.58)^2}{0.99} + \frac{(0.8)^2}{1} \approx \underline{\underline{1.5651}}$$

5.3.1. $A(s) = s^6 + 3s^5 + 5s^4 + 12s^3 + 6s^2 + 9s + 1$

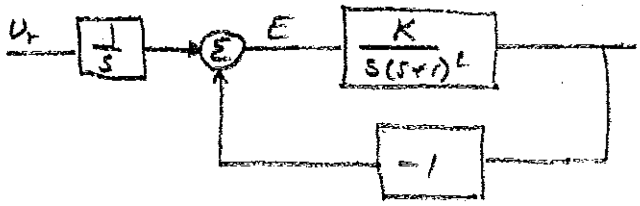
$B(s) = 3s^5 + s^4 + 12s^3 + 3s^2 + 9s + 1$

							α_k								β_k	
1	3	5	12	6	9	1			3	1	12	3	9	1		
<u>3</u>	0	12	0	9	0	0	$\frac{1}{3}$		3	0	12	0	9	0	1	
	3	1	12	3	9	1				1	0	3	0	1		
	<u>1</u>	0	3	0	1	0	3			1	0	3	0	1	1	
		1	3	3	6	1					0	0	0	0		
		<u>3</u>	0	6	0	0	$\frac{1}{3}$				3	0	6	0	0	
			3	1	6	1						0	0	0		
			<u>1</u>	0	1	0	3					1	0	1	0	
				1	3	1							0	0		
				<u>3</u>	0	0	$\frac{1}{3}$	$=\alpha_2$					3	0	0	$=\beta$
					3	1								0		
					<u>1</u>	0	3	$=\alpha_1$						1	0	$=\beta$
						<u>1</u>								0		

$$I = \sum_{k=1}^{6} \frac{\beta_k^2}{2\alpha_k} = \frac{1}{2}\left(\frac{1}{3} + 3\right) = \underline{\underline{\frac{10}{6}}}$$

<u>Se även lösning på dator!</u>

5.3.2. vill ha



$$E = \frac{1}{s}U_r - \frac{K}{s(s+1)^2}E$$

$$E = \frac{\frac{1}{s}}{1+\frac{K}{s(s+1)^2}}U_r = \frac{(s+1)^2}{s(s+1)^2+K}U_r = \frac{B(s)}{A(s)}U_r$$

$$\Phi_e(\omega) = \frac{B(i\omega)B(-i\omega)}{A(i\omega)A(-i\omega)}\Phi_{u_r} = \frac{1}{2\pi}\frac{B(i\omega)B(-i\omega)}{A(i\omega)A(-i\omega)}$$

$$r_e(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{B(i\omega)B(-i\omega)}{A(i\omega)A(-i\omega)}d\omega = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{B(s)\,B(-s)}{A(s)\,A(-s)}ds$$

$$A(s) = s(s^2+2s+1)+K = s^3+2s^2+s+K$$

$$B(s) = s^2+2s+1$$

				α_k
1	2	1	K	
$\underline{2}$	0	K	0	$\frac{1}{2}$
	2	$1-\frac{K}{2}$	K	
	$\underline{1-\frac{K}{2}}$	0	0	$\frac{4}{2-K}$
		$1-\frac{K}{2}$	K	
		$\underline{K}$	0	$\frac{2-K}{2K}$
			K	

			β_k
1	2	1	
2	0	K	$\frac{1}{2}$
	2	$1-\frac{1}{2}K$	
	$1-\frac{1}{2}K$	0	$\frac{4}{2-K}$
		$1-\frac{1}{2}K$	
		K	$\frac{2-K}{2K}$

Krav för stabilitet: $0 < K < 2$

$$I = \sum_{k=1}^{3}\frac{\beta_k^2}{2\alpha_k} = \frac{1}{2}\left(\frac{\left(\frac{2-K}{2K}\right)^2}{\frac{2-K}{2K}} + \frac{\left(\frac{4}{2-K}\right)^2}{\frac{4}{2-K}} + \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}}\right) =$$

$$= \frac{1}{2}\left(\frac{2-K}{2K} + \frac{4}{2-K} + \frac{1}{2}\right) = \frac{1}{2}\cdot\frac{(2-K)^2 + 8K + K(2-K)}{2K(2-K)} =$$

$$= \frac{4-4K+K^2+8K+2K-K^2}{4K(2-K)} = \frac{4+6K}{4K(2-K)} = \frac{2+3K}{2K(2-K)}$$

$$\frac{dI}{dK} = \frac{2K(2-K)\cdot 3 - (2+3K)(2(2-K)-2K)}{4K^2(2-K)^2} =$$

$$= \frac{12K-6K^2-(2+3K)(4-4K)}{4K^2(2-K)^2} = \frac{12K-6K^2-8-4K+12K^2}{4K^2(2-K)^2} = 0$$

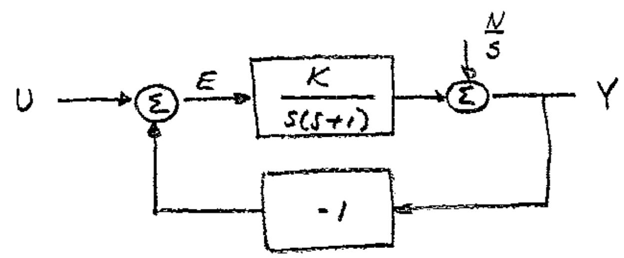
$$6K^2+8K-8=0$$

$$K^2+\frac{4}{3}K-\frac{4}{3}=0$$

$$\underline{\underline{K}} = -\frac{2}{3} \pm\sqrt{\frac{4}{9}+\frac{4}{3}} = -\frac{2}{3}\pm\sqrt{\frac{16}{9}} = -\frac{2}{3} \underset{(-)}{+} \frac{4}{3} = \underline{\underline{\frac{2}{3}}}$$

$$\underline{\underline{\Gamma(0)}} = I = \frac{2+2}{\frac{4}{3}\left(\frac{4}{3}\right)} = \frac{4\cdot 9}{16} = \underline{\underline{\frac{9}{4}}}$$

5.3.3.



$$\Phi_u = \frac{1}{\omega^2 + a^2} \qquad \Phi_n = b^2$$

$$E = U - Y = U - \left(\frac{1}{s}N + \frac{K}{s(s+1)}E\right) = \frac{U - \frac{1}{s}N}{1 + \frac{K}{s(s+1)}} = \underbrace{\frac{s(s+1)}{s(s+1)+K}}_{G_1(s)} U - \underbrace{\frac{s+1}{s(s+1)+K}}_{G_2(s)} N$$

$$\Phi_e(\omega) = G_1(i\omega)\,G_1(-i\omega)\,\frac{1}{(a+i\omega)}\cdot\frac{1}{(a-i\omega)} + G_2(i\omega)\,G_2(-i\omega)\,b^2$$

$$r_e(0) = 2\pi\Bigg(\underbrace{\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} G_1(s)\,G_1(-s)\,\frac{1}{(a+s)(a-s)}\,ds}_{I_1} + \underbrace{\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} G_2(s)\,G_2(-s)\,b^2\,ds}_{I_2}\Bigg)$$

$\underline{I_1}$: $A(s) = (s(s+1)+K)(a+s) = (s^2+s+K)(s+a) = s^3 + (1+a)s^2 + (K+a)s + Ka$

$B(s) = s(s+1) = s^2 + s$

				α_k				β_k
1	$1+a$	$K+a$	Ka		1	1	0	
$\underline{1+a}$	0	Ka	0	$\frac{1}{1+a}$	$1+a$	0	Ka	$\frac{1}{1+a}$
	$1+a$	$\frac{K+a+a^2}{1+a}$	Ka			1	$-\frac{Ka}{1+a}$	
	$\underline{\frac{K+a+a^2}{1+a}}$	0	0	$\frac{(1+a)^2}{K+a+a^2}$		$\frac{K+a+a^2}{1+a}$	0	$\frac{1+a}{K+a+a^2}$
		$\frac{K+a+a^2}{1+a}$	Ka				$-\frac{Ka}{1+a}$	
		$\underline{Ka}$	0	$\frac{K+a+a^2}{Ka(1+a)}$			Ka	$-\frac{1}{1+a}$
			Ka					

$$I_1 = \sum_{k=1}^{3} \frac{\beta_k^2}{2\alpha_k} = \frac{1}{2}\left(\frac{\frac{1}{(1+a)^2}}{\frac{K+a+a^2}{Ka(1+a)}} + \frac{\left(\frac{1+a}{K+a+a^2}\right)^2}{\frac{(1+a)^2}{K+a+a^2}} + \frac{\frac{1}{(1+a)^2}}{\frac{1}{1+a}} \right) =$$

$$= \frac{1}{2}\left(\frac{Ka}{(1+a)(K+a+a^2)} + \frac{1}{K+a+a^2} + \frac{1}{1+a} \right)$$

I_2: $A(s) = s(s+1) + K = s^2 + s + K$

$B(s) = b(s+1) = bs + b$

			α_k			β_k
1	1	K		b	b	
$\underline{1}$	0	0	1	1	0	b
	1	K			b	
	$\underline{K}$	0	$\frac{1}{K}$		K	b/K
		K				

$$I_2 = \frac{1}{2}\left(\frac{\frac{b^2}{K^2}}{\frac{1}{K}} + \frac{b^2}{1} \right) = \frac{b^2}{2}\left(\frac{1}{k} + 1\right)$$

$$\underline{\mathrm{Var}\, E = 2\pi\left(\frac{b^2}{2k} + \frac{b^2}{2} + \frac{Ka}{2(1+a)(K+a+a^2)} + \frac{1}{2(K+a+a^2)} + \frac{1}{2(1+a)} \right)}$$

$$\frac{\partial V}{\partial K} = 2\pi\left(-\frac{b^2}{2k^2} + \frac{2a(1+a)(K+a+a^2) - Ka\cdot 2(1+a)}{4(1+a)^2(K+a+a^2)^2} - \frac{1}{2(K+a+a^2)^2} \right) = 0$$

$$-\frac{b^2}{2k^2} + \frac{a(K+a+a^2-K)}{2(1+a)(K+a+a^2)^2} - \frac{1}{2(K+a+a^2)^2} = 0$$

$$-b^2(1+a)(K+a+a^2)^2 + \qquad (a^2+a^3)k^2 - (1+a)k^2 = 0$$

<u>Ger k.</u>

5.3.9. $$I = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{1}{A(s)\,A(-s)}\, ds$$

$$A(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_n$$

$$B(s) = 1$$

$$a_0^n \quad a_1^n \quad \dots \quad \alpha_k \qquad 0 \quad 0 \dots 0 \quad 1 \quad \beta_k$$

$$\beta_k = \frac{b_1^k}{a_1^k} = \begin{cases} 0 & k \neq n \\ \frac{1}{a_1^n} & k = n \end{cases}$$

$$\alpha_n = \frac{a_0^n}{a_1^n}$$

$$I = \sum_{k=1}^{n} \frac{\beta_k^2}{2\alpha_k} = \frac{1}{2}\frac{\beta_n^2}{\alpha_n} = \frac{a_1^n}{2(a_1^n)^2 a_0^n} = \underline{\underline{\frac{1}{2a_0^n a_1^n}}}$$

5.4.1.

$$x(t+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

$$e(t) \in N(0,1) \qquad E\,x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \operatorname{cov}[x(0), x(0)] = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

$$\hat{x}(t+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \hat{x}(t) + K(y(t) - \theta \hat{x}(t))$$

$$R_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P(t+1) = \Phi P(t) \Phi^T + R_1 - \Phi P(t) \theta^T [\theta P(t) \theta^T]^{-1} \theta P(t) \Phi^T$$

$$P(0) = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

$$K(t) = \Phi P(t) \theta^T (\theta P(t) \theta^T)^{-1} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \frac{1}{p_1} =$$

$$= \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_3 \end{bmatrix} \cdot \frac{1}{p_1} = \begin{bmatrix} p_1 + h p_3 \\ p_3 \end{bmatrix} \cdot \frac{1}{p_1} = \begin{bmatrix} 1 + h\, p_3/p_1 \\ p_3/p_1 \end{bmatrix}$$

$$P(t+1) = [\Phi - K(t)\theta] P(t) \Phi^T + R_1 = \left(\begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 + h\, p_3/p_1 \\ p_3/p_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right) \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} 1 & 0 \\ h & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -h\, p_3/p_1 & h \\ -p_3/p_1 & 1 \end{bmatrix} \begin{bmatrix} p_1 + h p_2 & p_2 \\ p_3 + h p_4 & p_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} p_1(t+1) = -h p_3 - h^2 \dfrac{p_2 p_3}{p_1} + h p_3 + h^2 p_4 = -h^2 \dfrac{p_2 p_3}{p_1} + h^2 p_4 \\[2ex] p_2(t+1) = -h \dfrac{p_2 p_3}{p_1} + h p_4 \\[2ex] p_3(t+1) = -p_3 - h \dfrac{p_2 p_3}{p_1} + p_3 + h p_4 \\[2ex] p_4(t+1) = -\dfrac{p_2 p_3}{p_1} + p_4 + 1 \end{cases}$$

$P_2 = P_3$

$$\begin{cases} P_1(t+1) = -h^2 \dfrac{P_2^2}{P_1} + h^2 P_4 & P_1(0) = \sigma_1^2 \\ P_2(t+1) = -h \dfrac{P_2^2}{P_1} + h P_4 & P_2(0) = 0 \\ P_4(t+1) = -\dfrac{P_2^2}{P_1} + P_4 + 1 & P_4(0) = \sigma_2^2 \end{cases}$$

$$\begin{cases} h P_4(t+1) = P_2(t+1) + h \\ h P_2(t+1) = P_1(t+1) \end{cases}$$

$$\underline{P_4(t+1)} = -\frac{P_2^2}{h P_2} + P_4 + 1 = -\frac{P_2}{h} + P_4 + 1 = -P_4 + 1 + P_4 + 1 = \underline{2} \qquad t \geq 1$$

$$\underline{P_2(t+1)} = -P_2 + h P_4 = h - h P_4 + h P_4 = \underline{h} \qquad t \geq 1$$

$$\underline{P_1(t+1)} = h P_2(t+1) = \underline{h^2} \qquad t \geq 1$$

$$\underline{K(0)} = \begin{bmatrix} 1 + h \dfrac{P_3(0)}{P_1(0)} \\ P_3(0)/P_1(0) \end{bmatrix} = \begin{bmatrix} 1+0 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}}$$

$$\underline{\underline{K(1)}} = \begin{bmatrix} 1 + h P_3(1)/P_1(1) \\ P_3(1)/P_1(1) \end{bmatrix} = \begin{bmatrix} 1 + h^2\sigma_2^2/h^2\sigma_2^2 \\ h\sigma_2^2/h^2\sigma_2^2 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \\ 1/h \end{bmatrix}}}$$

$$\underline{\underline{K(t)}} = \begin{bmatrix} 1 + h^2/h^2 \\ h/h^2 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \\ 1/h \end{bmatrix}}} \qquad t \geq 2$$

5.4.2.
$$\begin{cases} x(t+1) = \Phi x(t) + \Gamma u(t) + v(t) \\ y(t) = \Theta x(t) + e(t) \end{cases}$$

$$E\, v(t) v^T(s) = \delta_{s,t}\, R_1 \qquad E\, v(t) e^T(s) = \delta_{s,t}\, R_{12} \qquad E\, e(t) e^T(s) = \delta_{s,t}\, R_2$$

$$\hat{x}(t+1) = \Phi \hat{x}(t) + \Gamma u(t) + K\big(y(t) - \Theta \hat{x}(t)\big)$$

$$\tilde{x}(t+1) = (\Phi - K\Theta)\tilde{x}(t) + v(t) - K e(t)$$

$$P(t+1) = E\left[\tilde{x}(t+1) - E\tilde{x}(t+1)\right]\left[\tilde{x}(t+1) - E\tilde{x}(t+1)\right]^T =$$

$$= E\left[(\Phi - K\Theta)\tilde{x} + v - Ke\right]\left[(\Phi - K\Theta)\tilde{x} + v - Ke\right]^T =$$

$$= E\left[(\Phi - K\Theta)\tilde{x} + v - Ke\right]\left[\tilde{x}^T(\Phi - K\Theta)^T + v^T - e^T K^T\right] =$$

$$= (\Phi - K\Theta)P(t)(\Phi - K\Theta)^T + R_1 + K R_2 K^T - R_{12} K^T - K R_{21} =$$

$$= \Phi P(t)\Phi^T + R_1 + K(-\Theta P(t)\Phi^T - R_{21}) + (-\Phi P(t)\Theta^T - R_{12})K^T + K(\Theta P(t)\Theta^T + R$$

$$= \Phi P(t)\Phi^T + R_1 + \left\{K - [\Phi P(t)\Theta^T + R_{12}][\Theta P(t)\Theta^T + R_2]^{-1}\right\}[\Theta P(t)\Theta^T + R_2]\cdot$$

$$\cdot\left\{K - [\Phi P(t)\Theta^T + R_{12}][\Theta P(t)\Theta^T + R_2]^{-1}\right\}^T - (\Phi P(t)\Theta^T + R_{12})(\Theta P(t)\Theta^T + R_2)^{-1}\cdot$$

$$(\Phi P(t)\Theta^T + R_{12})^T.$$

$\Rightarrow$ Välj $K(t) = (\Phi P(t)\Theta^T + R_{12})(\Theta P(t)\Theta^T + R_2)^{-1}$

$$\Rightarrow P(t+1) = \Phi P(t)\Phi^T + R_1 - (\Phi P(t)\Theta^T + R_{12})(\Theta P(t)\Theta^T + R_2)^{-1}(\Phi P(t)\Theta^T + R$$

$$= \Phi P(t)\Phi^T + R_1 - K(t)(\Theta P(t)\Theta^T + R_2)K^T(t)$$

5.5.1. $dx_1 = x_2\,dt$

$dx_2 = dv$

$dy = x_1\,dt + de$

$$dx = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x\,dt + \begin{pmatrix} 0 \\ 1 \end{pmatrix} dv$$

$$dy = \begin{bmatrix} 1 & 0 \end{bmatrix} x\,dt + de$$

$$K(t) = P(t)C^T R_2^{-1} = \begin{pmatrix} p_1 & p_2 \\ p_2 & p_3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \frac{1}{r} = \frac{1}{r} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

$$\frac{dP}{dt} = AP + PA^T + R_1 - PC^T R_2^{-1} CP = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} p_1 & p_2 \\ p_2 & p_3 \end{pmatrix} + \begin{pmatrix} p_1 & p_2 \\ p_2 & p_3 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} p_1 & p_2 \\ p_2 & p_3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{1}{r} (1 \quad 0) \begin{pmatrix} p_1 & p_2 \\ p_2 & p_3 \end{pmatrix} = \begin{pmatrix} p_2 & p_3 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} p_2 & 0 \\ p_3 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$- \frac{1}{r}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}(p_1 \quad p_2) = \begin{pmatrix} 2p_2 & p_3 \\ p_3 & 1 \end{pmatrix} - \frac{1}{r}\begin{pmatrix} p_1^2 & p_1 p_2 \\ p_1 p_2 & p_2^2 \end{pmatrix} = \begin{pmatrix} 2p_2 - \frac{p_1^2}{r} & p_3 - \frac{p_1 p_2}{r} \\ p_3 - \frac{p_1 p_2}{r} & 1 - \frac{p_2^2}{r} \end{pmatrix}$$

$$\begin{cases} \dot p_1 = -\dfrac{p_1^2}{r} + 2p_2 \\[2ex] \dot p_2 = -\dfrac{p_1 p_2}{r} + p_3 \\[2ex] \dot p_3 = 1 - \dfrac{p_2^2}{r} \end{cases}$$

<u>Stationärt:</u>

$$\begin{cases} p_1^2 = 2r p_2 \\ p_1 p_2 = r p_3 \\ p_2^2 = r \end{cases} \Rightarrow \begin{cases} p_1 = \sqrt{2}\, r^{\frac{3}{4}} \\ p_2 = \sqrt{r} \\ p_3 = \sqrt{2}\, r^{\frac{1}{4}} \end{cases}$$

$$\underline{\underline{K = \begin{pmatrix} \sqrt{2}\, r^{-\frac{1}{4}} \\ r^{-\frac{1}{2}} \end{pmatrix}}}$$

$$\underline{\underline{P = \begin{pmatrix} \sqrt{2}\, r^{\frac{3}{4}} & r^{\frac{1}{2}} \\ r^{\frac{1}{2}} & \sqrt{2}\, r^{\frac{1}{4}} \end{pmatrix}}}$$

5.5.2. $\begin{cases} dx = Ax\,dt + Bu\,dt + dv \\ dy = Cx\,dt + de \end{cases}$

$$E\begin{pmatrix} dv \\ de \end{pmatrix}\left[dv^T \; de^T\right] = \begin{bmatrix} R_1 & R_{12} \\ R_{12}^T & R_2 \end{bmatrix} dt$$

$$\tilde{x} = x - \hat{x}$$

$$d\tilde{x} = Ax\,dt + Bu\,dt + dv - A\hat{x}\,dt - Bu\,dt - K(Cx\,dt + de - C\hat{x}\,dt) =$$

$$= (A - KC)\tilde{x}\,dt + dv - K\,de$$

$$\frac{dP}{dt} = (A-KC)P + P(A-KC)^T + R_1 + KR_2K^T - R_{12}K^T - KR_{12}^T =$$

$$= AP + PA^T + R_1 - (KCP + PC^TK^T - KR_2K^T + R_{12}K^T + KR_{12}^T)$$

$$= AP + PA^T + R_1 + KR_2K^T - K(CP + R_{12}^T) - (PC^T + R_{12})K^T$$

Fortsättning följer ej.

5.5.3. $\left.\begin{array}{l} y = \theta + n \\ \dot\theta = u \\ u = -ky \end{array}\right\} \Rightarrow d\theta = -k\theta dt + k dv$

$\theta(0) \in N(0, \sigma)$

$E(dv \cdot dv) = r$

$$\begin{cases} \dot P = -kP - Pk + k^2 r = -2kP + k^2 r \\ P(0) = \sigma^2 \end{cases}$$

<u>Se lemma 5.1!</u>

$\dot P = AP + PA^T + R_1 + KR_2K^T - KCP - PC^TK^T$

$\Rightarrow A = 0, \quad R_1 = 0, \quad R_2 = r, \quad C = 1$

$\Rightarrow \underline{K(t)} = PC^T R_2^{-1} = \underline{\frac{1}{r} P(t)}$

$\dot P = -\frac{2}{r} P^2 + \frac{1}{r} P^2 = -\frac{1}{r} P^2$

$\frac{\dot P}{P^2} = -\frac{1}{r} \quad \Rightarrow \left[-\frac{1}{P(t)} \right]_0^t = -\frac{1}{r} t$

$\frac{1}{\sigma^2} - \frac{1}{P(t)} = -\frac{t}{r} \quad \Rightarrow \underline{P(t)} = \frac{1}{\frac{1}{\sigma^2} + \frac{t}{r}} = \underline{\frac{r\sigma^2}{r + \sigma^2 t}} \to 0, \ t \to \infty$

<u>Konstant k:</u> $\dot P = -2kP + k^2 r$

$\underline{P(t)} = Ae^{-2kt} + \frac{kr}{2} = \underline{\left(\sigma^2 - \frac{kr}{2}\right) e^{-2kt} + \frac{kr}{2}} \to \frac{kr}{2}, \ t \to \infty$