1.2.1. $\begin{cases} \dot{x} = u \\ x(0) = 1 \end{cases}$

Minimera $J = \int_0^\infty (x^2 + u^2)\,dt = \int_0^\infty (x^2 + \dot{x}^2)\,dt$

$$\int_0^T (x(t) + \dot{x}(t))^2 dt = \int_0^T \left(x^2(t) + \dot{x}^2(t) + 2x(t)\dot{x}(t) \right) dt = \int_0^T (x^2(t) + \dot{x}^2(t))\,dt +$$

$$+ \left[x^2(t) \right]_0^T = \int_0^T \left(x^2(t) + \dot{x}^2(t) \right) dt + x^2(T) - x^2(0)$$

$$\Rightarrow J = x^2(0) - x^2(\infty) + \int_0^\infty (x(t) + \dot{x}(t))^2 dt$$

J begränsad $\Rightarrow x(\infty) = 0$

$\Rightarrow J \leq x^2(0) = 1.$

$$\left.\begin{array}{l} x(t) + \dot{x}(t) = 0 \\ x(0) = 1 \end{array}\right\} \Rightarrow x(t) = e^{-t}$$

$J = 1 \Rightarrow$ 1 eller 2

1 $\underline{u(t)} = \dot{x}(t) = \underline{-e^{-t}}$

2 $\underline{u(t) = -x(t)}$

1.2.2. Modell: $\dot{x} = au$

Verkligt system: $\dot{x} = u$

$$J_m = \int_0^\infty \left(x^2(t) + u^2(t)\right)dt = \int_0^\infty \left(x^2(t) + \frac{1}{a^2}\dot{x}^2(t)\right)dt = \int_0^\infty \left(x^2 + \frac{1}{a^2}\dot{x}^2 + 2x\dot{x}/a - 2x\dot{x}/a\right)dt =$$

$$= \int_0^\infty \left(x(t) + \frac{1}{a}\dot{x}(t)\right)^2 dt + \frac{1}{a}x(0) - \frac{1}{a}x(\infty)$$

① $u(t) = -x(t) \quad \Rightarrow \underline{J = 1}$

② $u(t) = -e^{-at}$

$$x(t) = 1 - \int_0^t e^{-as}ds = 1 + \left[\frac{1}{a}e^{-as}\right]_0^t = 1 + \frac{1}{a}\left(e^{-at} - 1\right) = \frac{a - 1 + e^{-at}}{a}$$

$$J = \int_0^\infty \left(x(t)^2 + u^2(t)\right)dt = \int_0^\infty \left(\frac{(a-1+e^{-at})^2 + e^{-2at}}{a^2}\right)dt =$$

$$= \int_0^\infty \frac{(a-1)^2 + 2(a-1)e^{-at} + 2e^{-2at}}{a^2}dt = \left[\frac{(a-1)^2}{a^2}t - \frac{2(a-1)}{a^2}e^{-at} - \frac{e^{-2at}}{a^2}\right]_0^\infty$$

$$J = \lim_{t\to\infty} \frac{(a-1)^2}{a^2}t + \frac{2(a-1)}{a^2} + \frac{1}{a^2} = \underline{\underline{\infty}}$$

1.3.1. $x(t) = a\cos t$

$\underline{x(t+1)} = a\cos(t+1) = a(\cos t\cos 1 - \sin t\sin 1) = x(t)\cdot\cos 1 - \sin t\sin 1$

$= x(t)\cos 1 - \cos(t - \pi/2)\sin 1 = \underline{x(t)\cos 1 - x(t-\pi/2)\sin 1}$

$x(t_0) = a\cos t_0 \Rightarrow a = \dfrac{x(t_0)}{\cos t_0}$

$\underline{x(t)} = a\cos t = \underline{\dfrac{x(t_0)}{\cos t_0}\cos t}$

1.3.2. $\begin{cases} x(t+1) = a x(t) + e(t) \\ \hat{x}(t+1) = a x(t) \end{cases}$

$E(x(t+1) - \hat{x}(t+1))^2 = E(a x(t) + e(t) - a x(t))^2 = E(e(t))^2 = \sigma^2$