Exercise 2

1. Problem 5.3 in the course book
2. Problems 5.4 and 5.5 in the course book
3. † Consider the transfer matrix
$$P(s) = P_1(s) + P_2(s),$$
where $P_1(s)$ is a proper rational transfer matrix and $P_2 \in \mathcal{RH}^\infty$. Let $P_1(s) = M_1(s)^{-1}N_1(s)$ be a left coprime factorization of $P_1(s)$. Construct a left coprime factorization $P(s) = M(s)^{-1}N(s)$ and prove that that it is indeed coprime.
4. Consider the following transfer matrices:
$$G_1(s) = \begin{bmatrix} \frac{6}{(s+2)(s+3)} & \frac{3}{(s+1)(s+3)} \\ -\frac{2}{s+2} & \frac{2}{(s+1)(s+2)} \end{bmatrix} \text{ and } G_2(s) = \begin{bmatrix} \frac{s+5}{s+4} & \frac{s+1}{s+4} \\ \frac{1}{s+4} & \frac{s+1}{s+4} \end{bmatrix}.$$
   - Construct their minimal state-space realizations and calculate their poles, invariant zeros, and their directions using the state-space formulae.
   - Construct the state-space realization of $G_2(s)G_1(s)$ using the state-space formula provided in the lecture. Calculate its poles, invariant zeros, and their directions.
   - Is this realization minimal? Explain why. If the realization is not minimal, construct a minimal realization and calculate, using this realization, the poles and transmission zeros of the transfer matrix $G_2(s)G_1(s)$

   (You may use Matlab for all necessary calculations.)
5. † Prove the state-space formulae for the inverse of bi-proper transfer matrix and for the "partial fraction expansion". (See page 17 in the second lecture.)
6. Consider the following state-space realizations:
$$G_1(s) = \left[\begin{array}{c|c} A & B_1 \\ \hline C & D_1 \end{array}\right], \quad G_2(s) = \left[\begin{array}{c|c} A & B_2 \\ \hline C & D_2 \end{array}\right], \quad G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C & D_1 & D_2 \end{array}\right].$$
   Prove that:
   - $G(s) = \begin{bmatrix} G_1(s) & G_2(s) \end{bmatrix}$,
   - if the realizations of G_1 and G_2 are minimal, then so is that of G,
   - the minimality of G does not necessarily imply that of G_2.
7. For the state-space formulae of doubly coprime factorization provided in the lecture (see page 19), show that indeed $G(s) = N(s)M^{-1}(s)$.