

Poles and transmission zeros of  $G_1$

	alg. multiplicity	geom. multiplicity	inp. direction	out. direction
poles				
-1	1	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$
-2	2	2	$\mathbb{R}^2$	$\mathbb{R}^2$
-3	1	1	$\begin{bmatrix} -0.97 \\ -0.24 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
zeros				
-4	1	1	$\begin{bmatrix} -0.32 \\ 0.95 \end{bmatrix}$	$\begin{bmatrix} -0.32 \\ 0.95 \end{bmatrix}$

Poles and transmission zeros of  $G_2$

	alg. multiplicity	geom. multiplicity	inp. direction	out. direction
poles				
-4	1	1	$\begin{bmatrix} -0.32 \\ 0.95 \end{bmatrix}$	$\begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}$
zeros				
-1	1	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.24 \\ -0.97 \end{bmatrix}$

Poles and invariant zeros of the realization of  $G_2G_1$  obtained using the state-space formulae for product.

(This realization is not minimal, since there is a pole-zero cancellation at  $-4$ .)

	alg. multiplicity	geom. multiplicity	inp. direction	out. direction
poles				
-1	1	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.97 \\ 0.24 \end{bmatrix}$
-2	2	2	$\mathbb{R}^2$	$\mathbb{R}^2$
-3	1	1	$\begin{bmatrix} -0.97 \\ -0.24 \end{bmatrix}$	$\begin{bmatrix} 0.89 \\ 0.45 \end{bmatrix}$
-4	1	1	$\begin{bmatrix} -0.98 \\ -0.18 \end{bmatrix}$	$\begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}$
zeros				
-4	1	1	$\begin{bmatrix} -0.32 \\ 0.95 \end{bmatrix}$	$\begin{bmatrix} 0.69 \\ -0.72 \end{bmatrix}$
-1	1	1	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.24 \\ -0.97 \end{bmatrix}$

Poles and transmission zeros of  $G_2G_1$

	alg. multiplicity	geom. multiplicity	inp. direction	out. direction
poles				
-1	1	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.97 \\ 0.24 \end{bmatrix}$
-2	2	2	$\mathbb{R}^2$	$\mathbb{R}^2$
-3	1	1	$\begin{bmatrix} -0.97 \\ -0.24 \end{bmatrix}$	$\begin{bmatrix} 0.89 \\ 0.45 \end{bmatrix}$
zeros				
-1	1	1	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.24 \\ -0.97 \end{bmatrix}$

Notice how  $G_2G_1$  inherits directions from the multipliers. Pay attention also for the role of directions in pole-zero cancellations.