Exercise 1

Introduction. The class of linear systems as a linear space. Norm and inner product as a way to measure distance. Banach and Hilbert spaces. The Lebesgue spaces L_2 and L_{∞} . The Hardy spaces H_2 and H_{∞} .

Reading Assignment

- Refresh in memory and elaborate the unknown details about necessary linear algebra and linear system facts in [Zhou] Ch. 2 and 3.
- Read [Zhou] Ch. 4. Optional reading: [Doyle,Francis,Tannenbaum] Ch. 2.
- 1. [Zhou] 2.5-2
- 2. Suppose that u is a continuous function whose derivative is continuous too. Which of the following qualifies as a norm for u?
 - (a) $\sup_t |\dot{u}(t)|$,
 - (b) $|u(0)| + \sup_{t} |\dot{u}(t)|$,
 - (c) $\max\{\sup_t |u(t)|, \sup_t |\dot{u}(t)|\},$
 - (d) $\sup_{t} |u(t)| + \sup_{t} |\dot{u}(t)|$.
- 3. Prove that the relation

$$\langle f, g \rangle_2 = \int_{-\infty}^{\infty} \operatorname{tr} \left[g(t)^* f(t) \right] dt$$

satisfies all axioms of the inner product.

- 4. [Zhou] 4.3
- 5. For the linear system y = Gu prove that

$$||G|| = \sup_{||u||=1} ||y||.$$

(Note the equality sign instead of inequality.)

6. Calculate the H_{∞} distance between two plants

$$G_1(s) = \frac{1}{s+1}, \qquad G_2(s) = \frac{1}{s+1}e^{-\theta s}$$

for $\theta = \{0.01, 0.1, 1\}$. How does the distance depend on θ ?

Hand-In problems:

- 1. [Zhou] 4.4
- 2. [Zhou] 4.8